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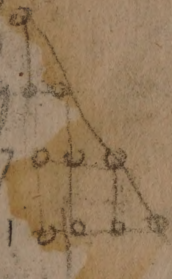

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RECREATIONS
FOR
GENTLEMEN and LADIES:

Being, ingenious

Sports and *Paſtimes.*

CONTAINING,

Many curious Inventions:
Pleaſant Tricks on the Cards and Dice:
Arithmetical Sports:
Diverting Experiments, natural and artificial:
Recreative Fire-Works:

AND

Other Curioſities, affording variety of Entertainment.

Tranſlated from the French of Monſ. OZANAM.

The FOURTH EDITION.

DUBLIN:

Printed by JAMES HOEY, Junior,
MDCCLIX.

TO THE

READER.

THE author of the following treatiſe, Monſieur Ozanam, *is a perſon ſo well known and deſervedly eſteemed, amongſt the learned who underſtand him in his native language, that if all others were alike acquainted with his worth, his name would be a ſufficient recommendation.*

This book is ſuch a collection of the moſt curious, moſt ſurprizing, moſt uſeful, and moſt agreeable performances of the arts and ſciences as may prove a ſpring of invention to the ingenious, furniſhing them with hints of innumerable other diſcoveries and contrivances, ſerviceable to the neceſſity, or the conveniency, or the pleaſure of human life. [Thus far the tranſlator.]

The Author's Account of the Work.

IT has been an opinion of long ſtanding, that there was ſome ſecret art amongſt the moſt learned of the Jews, of the Arabians, and of the diſciples of that antient academy, which was in Egypt when Moſes was there educated, and ſtill flouriſhed in the time of Solomon; inſomuch that it hath excited the curioſity of the fineſt wits to endeavour the diſcovery of it: but is it poſſible

to learn an art without a maſter, and without books? The learned of that time committed nothing to writing; or if they did, it was enigmatical, and ſo remote from what a reader did expect, that of them it may be ſaid, their ſilence was more inſtructive than their diſcourſes.

Father Schott ſaith there are three ſorts of Cabala, (ſo is that ſecret art of the Orientals called;) that of the Rabbies, that of Raimond Lully, and that of the Algebriſts. The firſt he knows not what it is; the two laſt are Recreations in numbers and figures: and no doubt is to be made, but the firſt is of the ſame ſort. Joſephus, who was a Levite, writes with confidence, that by right of his birth he had been inſtructed in all the myſteries of the Jews, and had been taught all the ſecrets of their art. He boaſted, from a courtly principle which ſwayed him more than his conſcience, that, by his art he had foretold the elevation of Titus to the Imperial dignity. He concealed his game as men of cunning ſhould, and as our maſters teach us. He gives out himſelf for a miraculous perſon; and when he relates the adventure where he ſhould have loſt his life by the deſpair of the ſoldiers, reſolved to cut one another's throat rather than ſurrender to the Romans, he attributes his deliverance to chance and a miracle. Notwithſtanding Hegeſippus, who wrote the ſame hiſtory, ſays, that Joſephus did that miracle by the knowledge of numbers and figures: for he made thoſe diſperadoes to be ranged in ſuch an order, that the lot fell upon thoſe, whom the commander deſired to have deſtroyed: he ſav'd his own life, not by reaſon of being a Levite, but becauſe he was a mathematician. Monſieur Bachet, in his 23d. probl. deſcribes this ſecret; who, had he then lived, would have been accounted as great a magician as Joſephus. Hence it appears, that the

moſt abſtracted knowledge may be reduced to practice, and what ſeems moſt remote may become of uſe.

'Tis moſt aſtoniſhing to find, that in the time of the emperours Diocleſian and Conſtantine, the mathematicks were prohibited by the laws, as a dangerous ſcience, under the ſame penalties as ſorcery or magick, being reputed equally criminal and pernicious to civil ſociety; as appears from the 17th title of the 9th book of Juſtinian's Code. No doubt this was an effect of the ignorance which at that time reigned; and becauſe of the great number of impoſtors who uſed the mathematicks to cheat and deceive the credulity of the illiterate. Nevertheleſs, the ſtupidity of thoſe is to be blamed, who ſuffered themſelves to be gull'd; and their negligence is not to be allowed, who will not ſufficiently improve their underſtanding, ſo as to be in a condition not to be abuſed. There have been ſtates wherein tricks and little thefts, cleverly performed, were permitted, that all might be on their guard, and accuſtomed to a requiſite precaution.

Ignorance keeps the world in perpetual admiration, and in a diffidence, which ever produces an invincible inclination to blame and perſecute thoſe that know any thing above the vulgar; who, being unaccuſtomed to raiſe their thoughts beyond things ſenſible, and unable to imagine that nature employeth agents that are inviſible and impalpable, aſcribe moſt an end to ſorceries and demons, all effects whereof they know not the cauſe. To remedy theſe inconveniencies is the deſign of theſe Recreations, and to teach all to perform theſe ſorceries which were dreaded by the councel of Juſtinian: and hereby will be vindicated the fame of Thomas Aquinas, Albertus Magnus, Solomon, and many

other

other great men, who had never been accufed for magicians, but becaufe they knew fomething more than others; more effectually than has been done by the learned, who have been fatisfied by dint of argument only, to plead their caufe.

It will, perhaps, be here objected, that by the paftimes of mind, prefented to the world in the enfuing book, the reader is diverted from that ftudy and application, to which he might have been engaged by treatifes of a ferious nature, which fix the thoughts, rendering them penetrating and inquifitive. To this it might fuffice to alledge the example of men famous for learning, whofe like practice in this matter, may feem a juftification beyond any other could be brought. The learned Bachet, Sieur de Meziriac, famous for his excellent works, began to make himfelf known to the learned world, by a collection which he intitled *pleafant problems performed by numbers*; by which he defigned to make trial of his own ability, and the opinion of the world, before he publifhed his commentaries on the arithmetick of Diophantus of Alexandria, and his other works by which he hath purchafed to himfelf immortal glory. Many orher authors of this age, as the famous father Kircher, the fathers Schott and Bettin, have gained no lefs renown by the diverting problems in their works, than by their reafonings, and more ferious obfervations.

But left thefe illuftrious men, adduced as precedents, fhould themfelves be expofed to the cenfure of thofe who would accufe them of novelty; inftances much more ancient, grounded on folid reafon, fhall be here produced, whereby it will appear that in all times this has been done by the greateft men; being perfuaded, that the fame fource of reafon that makes men take pleafure in admiration,

caufes

caufes them in like manner, to find delight in things which are the object of that paffion.

The enigmatical fentences and propofitions, fo much admired and promoted by the kings of Syria, which occafioned the continuance of the parabolical ftile fo long after, were nothing elfe but paftimes of mind, and entertainments equally fitted to excite pleafure, and to give enlargement of underftanding. Perfons of higher birth and rank were of the fame make at that time, as thofe of our own are now: what was painful and laborious did difcourage them: to engage them to ftudioufnefs and reflection, by pleafure and curiofity, was a piece of extraordinary fkill and dexterity. Doubtlefs, the education Nathan, by this means, gave to Solomon, did mightily conduce to that grandeur of foul, and to that admirable wifdom which conftitutes the character, and is the glory of that prince.

It was alfo by way of diverfion the Chaldeans and Egyptians, the inventers of aftronomy, did foretell to their friends the time, and other circumftances of eclipfes, and erected fyftems which fhewed the length of the days, demonftrated the courfe of the ftars, and reprefented all the varieties of the celeftial motions; being perfuaded no lefs than the Grecians, that the firft intellectual pleafures are thofe which proceed from mathematical fciences, in which they educated their children. They were convinced, that childrens reafon, though not yet in action, was not without its ftrength, and wanted only to be put in motion, in order to its progrefs towards perfection; which might be effected by exciting in them a curiofity, that would do the fame with them which a long train of neceffities does with thofe of more advanced years. Herein lay the fecret of Socrates, who taught children to

resolve the greatest difficulties of geometry and arithmetick: this was the key with which he laid open their understanding, knew its strength and predicted their destiny: this was instead of that Demon or Genius he is said to have consulted, and which is reported ever to have accompanied him.

Though these plays of the intellect, here spoken of, seem only amusements to pass away the time; yet are they possibly of no less advantage than those exercises in which the youths of quality are bred up at academies, which fashion as well as invigorate their bodies, and give them a graceful air in their deportment: for to be accustomed to discern the proportions, and the force of mixtures; to find out an unknown point required, amongst a confused infinity of others; to take a right method in resolving the most intricate and perplexing propositions; is to have the mind fitted for business, to be armed against surprizes, and prepared to overcome unexpected difficulties: things of no less consequence, one would think, than adjusting the motions of the body by the instructions of a dancing master, or the tone of a voice by that of a musician.

Besides, are not diversions sometimes necessary? And can any one be diverted by what he despises, or is ashamed of? Would a statesman choose to be performing at dancing matches, in the intervals of councils, and of important business? Or were it becoming for him to be found in those exercises wherein he spent the time of his youth? Decency, business, and health would in no wise allow it. But pastimes of mind are for all seasons and all ages: they instruct the youth and divert the old: they are not beneath the rich nor above the ability of the poor: they may be used by either sex without transgressing

greſſing the bounds of modeſty. Thoſe diverſions have this further and peculiar advantage, that there can be no exceſs in them: for ſeeing there is a regular conduct of reaſon therein, through all the ſteps it ſhould take, it cannot be conceived how it ſhould touch upon any extreme, its exerciſe being within the due medium, where the ſolution of the propoſed ſport is to be found.

Thoſe who have had the curioſity to obſerve the conduct of great men in their private actions, have found that they are diſtinguiſhed as well in their recreations as in their buſineſs. Auguſtus uſed to exerciſe himſelf in the evenings with his family at theſe diverſions, not judging it beneath him; and recorded with no leſs exactneſs the particulars of his recreations than thoſe of his important affairs. That learned lawyer Mutius Scevola, after his conſultations were over, diverted himſelf by playing at Cheſs, and became one of the beſt players of his time. Pope Leo X. one of the greateſt men of his age, played ſometimes at Cheſs, if we may believe Paulus Jovius, to recreate himſelf after the fatigues of buſineſs.

It is certain the game of Cheſs was invented for inſtruction as well as diverſion. The attacks and defences, the divers ſteps and advantages of the different pieces, may furniſh the conſiderate with political and moral reflections. By the diſaſter of of the king, we may learn, that a prince muſt infallibly fall under his enemies power when deprived of his ſoldiers; and that he cannot neglect the preſervation of them without expoſing himſelf and his dominions.

All games that are, or may be invented, may be reduced to three ranks. The firſt is of thoſe that depend altogether on numbers and figures; as the Cheſs, the Draughts, and ſome others: the ſecond

of

of thofe that are governed by chance; as the dice, and fuch like; the third fort is of thofe that are fubjected to the laws of motion, and require an exactnefs and regularity thereof; fuch as fhooting with guns, and with bows, the Tennis and Billiards. There are fome plays of a mixed nature, depending partly on fkill, partly on chance; as the Tables, the Cards and moft others. But it is certain, there is none of them which might not be fo far fubjected to the rules of the mathematicks, that one might be affured of the victory, had he but all the underftanding requifite. Games of dexterity depend fo much upon principles of ftaticks and mechanicks, that it is only the want of a due knowledge of their rules, or of the way of reducing them to practice, that makes a man fall fhort of conqueft.

In all plays of chance whatever, the victory depends upon the coming up of a certain number, upon weight, or upon the dimenfions of a figure. The gamefter that gives the motion, might at pleafure determine the end of it, were his fkill and dexterity perfect; and though this does not feem to be poffible, there being none to be found mafter of fo much cunning; yet it is true that this might be done, and that an infallible method of winning, at Chefs for inftance, is not abfolutely impoffible: but no body has hitherto found it out; nor perhaps ever will, feeing it depends on two great a number of combinations. It is enough that the point of perfection is poffible, to encourage the labour of the curious. "*A perfect orator*, faid Tully, *never* "*was, and yet is poffible.*" His picture drawn by that famous mafter, may be a pattern for the imitation of thofe who ftudy to excell in eloquence. The like may be faid of a poet, a painter, an architect, a phyfician, and all others. In like manner,

though

though it is true that no one has attained an infallible method in all plays, nor perhaps in any one; this ought to hinder none from endeavouring to become as ſkilful as he can, and to come up as near as may be to the idea of that method, which, becauſe founded upon principles of mathematicks, muſt participate of a mathematical certainty.

It may poſſibly be thought an extraordinary attempt to endeavour to proſelyte gameſters to this opinion, and to engage ſtateſmen and great commanders in the ſtudy of arithmetical recreations: notwithſtanding there can be no harm in carrying the light, let who will follow afrer it: yea, is it poſſible to hinder mankind from learning what is built on the moſt natural principles, and on truths flowing from the eſſence of things? ſhould they be deprived of pleaſure ſo inviting by their utility; and which are ſo familiar, ſo eaſy, and ſo ſuited to all endowed with reaſon, that to berave men of them, were to rob them of what is moſt agreeable in life.

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SPORTS

AND

PASTIMES.

Arithmetical Sports.

SPORT I.

Three jealous husbands with their wives being ready to paſs by night over a river, do find at the river ſide a boat which can carry but two perſons at once, and for want of a waterman they are neceſſitated to row themſelves over the river at ſeveral times: The queſtion is, how thoſe ſix perſons ſhall paſs 2 by 2, ſo that none of the 3 wives may be found in the company of 1 or 2 men, unleſs her huſband be preſent.

THEY muſt paſs in this manner, *viz.* firſt two women paſs, then one of them brings back the boat and repaſſes with the third woman; that done, one of the three women brings back the boat, and ſitting down npon the ground with her huſband, permits the other two men to paſs over to find their

wives: then one of the ſaid men with his wife brings back the boat, and placing her upon the ground, he takes the other man, and repaſſes with him; Laſtly, the woman who is found with the three men enters the boat, and at twice goes to fetch over the other two women.

SPORT II.

A Country-man having a fox, a gooſe, and a peck of corn, in his journey came to a river, where it ſo happened that he could carry but one over at a time. Now, as no two were to be together that might deſtroy each other; ſo he was at his wit's end how to diſpoſe of them; for, ſays he, tho' the corn can't eat the gooſe, nor the gooſe eat the fox, yet the fox can eat the gooſe, and the gooſe eat the corn: The queſtion is, how he muſt carry them over?

FIRST he muſt carry over the gooſe, leaving the fox and corn, (for the fox will not eat the corn) then, returning back, he may carry over the fox, bringing the gooſe back again; then leaving the gooſe, he may carry over the corn; laſtly, he muſt return to fetch the gooſe.

SPORT III.

Two merry companions are to have equal ſhares of eight gallons of wine, which are in a veſſel containing exactly eight gallons: now to make this equal partition they have only two other empty veſſels, of which one contains five gallons, and the other three; the queſtion is, how they ſhall exactly divide the wine by the help of thoſe three veſſels?

TO anſwer this queſtion; let us call the veſſel of 8 gallons A, the 5 gallon veſſel B, and the 3 gallon veſſel C. We ſuppoſe there are 8 gal-

lons of wine in the veſſel A, and the other two, B and C, are empty, as you ſee in D. Having filled the veſſel B with wine out of the veſſel A, in which there will then remain but 3 gallons, as you ſee at E; fill the veſſel C with wine out of the veſſel B, in which, by conſequence, there will then remain but 2 gallons, as you ſee at F. This done, pour the wine of the veſſel C into the veſſel A, where there will then be 6 gallons, as you ſee in G; and pour the 2 gallons of the veſſel B into the veſſel C, which will then have 2 gallons, as you ſee at H; then fill the veſſel B with wine out of the veſſel A, by which means there will remain but 1 gallon in it, as you ſee at I; and conclude the operation by filling the veſſel C with wine out of the veſſel B, in which there will then remain juſt 4 gallons, as you ſee at K; and ſo the queſtion is ſolved.

	8	5	3
	A	B	C
D	8	0	0
E	3	5	0
F	3	2	3
G	6	2	0
H	6	0	2
I	1	5	2
K	1	4	3

If, inſtead of the veſſel B, you would have the 4 gallons to remain in A, which we ſuppoſed to be filled with 8 gallons; fill the veſſel C with wine out of the veſſel A, and ſo there will remain but 5 gallons in it, as you ſee at D; pour the 3 gallons of the veſſel C into the veſſel B, which will then have 3 gallons of wine, as you ſee at E; and having again filled the veſſel C with wine out of the veſſel A, where there will then remain but 2 gallons, as you ſee at F; fill up the veſſel B with wine out of C, where there will then remain but

	8.	5.	3.
	A	B	C
	8.	0.	0.
D	5	0	3
E	5	3	0
F	2	3	3
G	2	5	1
H	7	0	1
I	7	1	0
K	4	1	3

1 gallon

1 gallon, as you ſee at G; at laſt, having poured the wine of the veſſel B into the veſſel A, where there will then be 7 gallons, as you ſee at H; pour the gallon of wine that is in C into the veſſel B, which by conſequence will have only 1 gallon, as you ſee at I; fill the veſſel C with wine out of the veſſel A, where there will then remain juſt 4 gallons, purſuant to the demand of the queſtion, as you ſee at K.

SPORT IV.

A blind Abbeſs, viſiting her Nuns, who were equally diſtributed in eight cells built at the four corners of a ſquare, and in the middle of each ſide; finds an equal number of perſons in each row or ſide containing three cells: At a ſecond viſit, ſhe finds the ſame number of perſons in each row, tho' their number was enlarged by the acceſſion of four men: And coming a third time, ſhe ſtill finds the ſame number of perſons in each row, tho' the four men were then gone, and had carried each of them a nun with them.

TO reſolve the firſt caſe, when the 4 men were got into the cells, we muſt conceive it ſo, that there was a man in each corner-cell, and that two nuns removed from thence to each of the middle cells: At this rate, each corner cell contained one perſon leſs than before, and each middle cell 2 more than before, ſuppoſe then that at the firſt viſitation, each cell contained 3 nuns, and ſo that there were 9 in each row, and 24 in all; at the ſecond viſit which is the firſt caſe in queſtion there muſt have been 5 nuns in each middle

3	3	3
3		3
3	3	3

2	5	2
5		5
2	5	2

middle cell, and 2 perſons, *viz.* 1 man and 1 nun in each corner cell; which ſtill makes 9 perſons in each row.

To account for the ſecond caſe, when the 4 men were gone, and 4 nuns with them; each corner cell muſt have contained 1 nun more than at the firſt viſit, and each middle cell 2 fewer: And thus, according to the ſuppoſition laid down, each corner cell contained 4 nuns, and there was only 1 in each middle cell; which ſtill make 9 in a row, though the whole number was but 20.

4	1	4
1		1
4	1	4

Note. Though at firſt ſight it may be thought by ſome that the four foregoing Sports cannot be reſolved by any certain rule, but only by many trials; yet by infallible argumentation and diſcourſe, the ſolution of thoſe queſtions may be found out, or elſe the impoſſibility of them, if by chance they ſhould be propounded impoſſible.

SPORT V.

Fifteen Chriſtians and fifteen Turks being at ſea in one and the ſame ſhip in a terrible ſtorm, and the pilot declaring the neceſſity of caſting the one half of thoſe perſons into the ſea, that the reſt might be ſaved; they all agreed, that the perſons to be caſt away ſhould be ſet out by lot after this manner, viz. *the thirty perſons ſhould be placed in a round form like a ring, and then beginning to count at one of the paſſengers, and proceeding circularly, every ninth perſon ſhould be caſt into the ſea, until of the thirty perſons there remained only fifteen. The queſtion is, how thoſe thirty perſons ought to be placed, that the*

lot might infallibly fall upon the fifteen Turks and not upon any of the fifteen Christians?

FOR the more eafy remembring of the rule to refolve this queftion, I fhall prefuppofe the five vowels, *a*, *e*, *i*, *o*, *u*, to fignify five numbers, to wit, (*a*) one, (*e*) two, (*i*) three, (*o*) four, and (*u*) five; then will the rule itfelf be briefly comprehended in thefe two following verfes:

> *From numbers, Aid, and Art,*
> *Never will fame depart.*

In which verfes you are principally to obferve the vowels, with their correfpondent numbers before affigned; and then beginning with the Chriftians, the vowel *o* (in *from*) fignifies that four Chriftians are to be placed together; next unto them the vowel *u* (in *Num.*) imports that five Turks are to be placed together; in like manner *e* (in *bers*) denotes two Chriftians; *a* (in *Aid*) one Turk; *i* (in *Aid*) three Chriftians; *a* (in *and*) one Turk; *a* (in *Art*) one Chriftian; *e* (in *ne*) two Turks; *e* (in *ver*) two Chriftians; *i* (in *will*) three Turks; a (in *Fame*) one Chriftian; *e* (in *Fame*) two Turks; *e* (in *de*) two Chriftians; *a* (in *part*) one Turk.

The invention of the faid rule, and fuch like, depends upon the fubfequent procefs, *viz.* If the number of perfons be thirty, let thirty figures or cyphers be placed circularly, or elfe in a right line as you fee,

ooooϕϕϕϕϕooϕoooϕoϕϕooϕϕooϕoooϕ

That done, begin to count from the firft, and mark the ninth (or what other fhall be affigned) by putting a point or crofs over it; then count forward from that which you have marked, and place another point over the next ninth; and continue to

do the ſame, beginning again when you ſhall be at the end (if the cyphers are placed in a right line,) and paſſing over thoſe which you had already marked, until you have marked the number required, as in the example propounded, until you have marked fifteen; for then all the cyphers marked ſhall be thoſe which muſt be caſt away, and the others thoſe that are to remain. Hence it is evident, that if you obſerve how thoſe cyphers marked, are diſpoſed among thoſe which are not marked, you will eaſily make a rule for any number whatſoever.

By this invention (as ſome conjecture) the famous hiſtorian, *Joſephus* the *Jew*, preſerved his life very ſubtilly in the cave, to which himſelf and forty of his countrymen had fled from the furious and conquering *Romans* at the ſiege of *Jotapata:* For his ſaid countrymen having moſt wickedly reſolved to kill one another, rather than yield to their enemies, he at length (when no arguments that he could uſe would diſſuade them from ſo horrid an act) prevailed with them to execute their tragical deſign by lot; and ſo by the help of the aforeſaid artifice, as we may ſuppoſe, himſelf, with one other perſon only remained alive, after the reſt were inhumanly murdered, they agreed to put an end to the lot, and thereby ſave their lives. This ſtory you may ſee at large in the fourteenth chapter of the third book of the Hiſtory of *Joſephus*, of the wars of the Jews.

SPORT VI.

To ſubſtract, with one ſingle operation, ſeveral ſums, from ſeveral other ſums given.

TO ſubſtract all the ſums that are under the line at B, from all the ſums above the line at A; begin by adding the numbers of figures of

56243	
84564	A
3252	
26848	
2942	
3654	B
2[illegible]08	
162003	

the right hand column under the line saying, 8 and 4 is 12, and 2 makes 14; which taken from the nearest tens, *viz.* 20, there remains 6; which we add to the correspondng column above, saying 6 and 8 make 14, and 2 is 16, and 4 make 20, and 3 make 23: here we write 3 underneath; and in regard there are just two tens, as before, we retain or carry nothing. This done, we add after the same manner, the numbers of the next lower column, saying, 0 and 5 is 5, and 4 make 9; which taken from the nearest ten, leaves 1; which we add, as above, to the superior correspondent column, saying, one and 4 make 5, and 5 make 10, and 6 make 16, and 4 make 20: here we set 0 underneath, and there being here two tens, whereas in the inferior corresponding column there was but one, we keep or carry the difference 1 to be taken from the next inferior column, because we found more tens in A than in B: For had we found fewer in A than in B, we must have added the difference: and if it should so fall out, that this difference can not be taken from the interior column, for want of significant figures, as it happens here in the fifth column; we must add it to the superior column, and write the whole Sum under the line. Thus in the example proposed, we have 162003, for the remainder of the substraction.

SPORT VII.

Compendious ways of Multiplication.

To multiply any number, 128 for instance by a number that's the product of the multiplication of two other numbers; 24 for instance, the product of the multiplication of 4 and 6, or of 3 and

and 8: we multiply the number propoſed 128 by 4, and the product 512 by 6, (or elſe 128 by 3, and the product by 8) and have 3072 for the required multiplication.

Hence it follows, that to multiply a number propoſed by a ſquare number, we muſt multiply the number propoſed by the ſide or the root of the ſquare, and then the product by the ſame ſide again. Thus to multiply 128 by 25, we multiply it by 5, and the product by 5 again.

To multiply any number, 128 for inſtance, by a number that's the product of the multiplication of *three* other numbers, as 108 the product of 2, 6, and 9, or of 3, 6, and 6: we multiply 128 by 2, the product by 6, and the ſecond product by 9; or elſe 128 by 3, the product by 6, and the ſecond product by 6.

The conſequence of this is, that to multiply any number propoſed, by a cube-number, we muſt multiply it firſt by the ſide or root of the cube; then the product of that multiplication by the ſame root, and the ſecond product by the ſide again. As, to multiply 128 by 125, the cube-root of which is 5, we multiply 128 by 5, and the product 640 by five again, and the ſecond product 3200 by five again. Thus to find how many cubical feet are in 32 cubical toiſes, we multiply 32 by 6, the product of that by 6, and the ſecond product by 6.

To multiply any number by what power you will of five, add to the number propoſed, on the right-hand, as many cyphers as the exponent of the power contains unites, as, one cypher for 5, two for its ſquare 25, three for its cube 125, and ſo on: and divide the numbers thus augmented by the like power from 2, that is, 2 for 5, 4 for its ſquare 25, 8 for its cube 125, and ſo on.

Thus to multiply 128 by 5, we divide 1280 by 2, and the quotient 640 is the product of the multipli-

tiplication: but to multiply 128 by 25 the ſquare of 5, we divide 12800 by 4 the ſquare of 2, and the quotient is the product demanded; and to multiply the ſame number 128 by 125 the cube of 5, we divide 128000 by 8 the cube of 2. And ſo on.

To know how many inches are in 53 foot, we multiply 53 by 12; or it might be done by multiplying 53 by 2, and the product by 6; or 53 by 3, and the product by 4. But there's a way of doing it without any multiplication; *viz.* by ſetting down 53 under 53, and then 53 again under both, advancing it a column to the left, ſo as to make 3 ſtand under 5; for the ſum of theſe three is 636, the number of inches contained in 53 foot, or of pence in 53 ſhillings.

```
 53
 53
53
———
636
```

To multiply together two numbers compoſed of ſeveral figures, 12, for inſtance, and 18; we reduce the firſt number 12, into theſe three parts, each of which conſiſts only of one figure, 2, 4, and 6, and in like manner the ſecond number, 18, into 4, 6, 8; each of which laſt muſt be multiplied by 2, the firſt part of the firſt number; and then by 4, the 2d figure of the ſame firſt number; and at laſt by 6, the third part: and the ſum of all theſe products anſwers the demand.

SPORT VIII.

Diviſion ſhortened.

TO divide a large number by a ſmaller, by only addition and ſubſtraction, as 1492992 by 432; we commonly put the diviſor to the left, under 1492, to know how many times 'tis contained in that number. But yet we may ſave ourſelves that labour by making a tariff of the diviſor; for which end we place it on the right over-againſt 1; then

then add it to itſelf, or double it, and place that over-againſt 2: Then we add it to the double, and place the ſum oppoſite to 3; adding it to the triple, we have its quadruple oppoſite to 4; as the additional of itſelf to the quadruple, gives the quintuple oppoſite to five; and ſo of the other multiples oppoſite to 6, 7, 8, 9, 10: The laſt of which, *viz.* the multiple correſponding to 10, ought, if the table is right done, to be the ſingle diviſor with a cypher on the right-hand.

1	432	1492992	(3456
2	864	1296...	
3	1296	——	
4	1728	1969	
5	2160	1728	
6	2592	——	
7	3024	2419	
8	3456	2160	
9	3888	——	
10	4320	2592	
		2592	
		——	
		000	

Having thus prepared your table, proceed in the common way of diviſion; and every time you have occaſion to know how often your diviſor is contained in the correſponding number, look in your table for the neareſt number that does not exceed; and the number to which that is oppoſite gives you at one view the figure you are to put in your quotient. As, in the beginning of the diviſion here exemplified, you want to know how often 432 is to be found in 1492; in your table, you find 1296 (the neareſt number to 1492 and not exceeding it) oppoſite to 3, and accordingly 3 is the firſt figure of your quotient; and ſo of all the reſt.

This

This way is very convenient, when we have occasion to divide large numbers by a ſmaller number; for the tariff of our diviſor keeps us from being at a ſtand, by reſolving us readily upon all our diviſions. This is frequently the caſe of ſurveyors of land, who have occaſion to divide large numbers by 144, when they want to reduce ſquare inches into ſquare feet; or by 1728, when they want to reduce cubical inches into cubical feet.

To divide any number by what power you will of 5, multiply it by the like power of 2, and cut off from the right hand of the product as many figures as there are unites in the degree of the power; the remaining figures on the left will repreſent the quotient of the diviſion, and thoſe ſtruck off will be the numerator of a fraction, the denominator of which will be the like power of 10.

To divide any number by a ſmaller, that is the product of the multiplication of two yet ſmaller numbers, divide the number propoſed, by one of the two ſmaller, and the quo ient by the other; and the ſecond quotient ariſing from the laſt diviſion, is what you want.

Thus to divide 20736 by 24, the product of 3, and 8, or of 4 and 6, we take the 8th part of its 3d, or the 6th part of its 4th, or, (which is the ſame thing) we take the 3d of its 8th part, or the 4th of its 6th, and our quotient proves 1728.

Hence to reduce ſquare feet to ſquare toiſes, (a toiſe is 6 foot) we muſt take the 6th part of the 6th part of the number propoſed of ſquare feet, becauſe a ſquare toiſe is 36 ſquare foot, and 6 times 6 is 36. Thus to reduce 542 ſquare feet to ſquare toiſes, we muſt take the 6th part of $90\frac{2}{6}$ (the 6th part of 542) and ſo have 15 ſquare toiſes and 2 ſquare feet, as the value of 542 ſquare feet.

SPORT

SPORT IX.

Curious properties of Numbers.

I. NUMBER 9 has this property; that when it multiplies any number of integers whatsoever, the ſum of the figures in the product is diviſible by 9: thus 53, multiplied by 9, makes the product 477; the figures of which, added together, *viz.* 7 and 7 and 4 make 18, which is exactly diviſible by 9.

II. Take any two numbers whatſoever, either one of the two, or their ſum, or their difference is diviſible by 3: thus, of the two numbers 6 and 5, 6 is diviſible by 3; of 11 and 5 the difference 6 is diviſible by 3; of 7 and 5 the ſum 12 is diviſible by 3.

III. The product ariſing from the multiplication of two numbers, the ſquares of which make a joint ſquare number, is diviſible by 6: thus 12 the product of 3 and 4, the ſquares of which, *viz.* 9 and 16, make together the ſquare number 25; this 12, I ſay, is diviſible by 6.

To find two numbers, the ſquares of which make together a ſquare number, multiply any two numbers, the one by the other, and the double of the product will be one of the two numbers demanded, and the difference of their ſquares will be the other. Thus in 2 and 3, the double of their product 12, and 5 the difference of their ſquares (4 and 9) are two numbers of that quality, that their ſquares 144 and 25 make together the ſquare number 169, the root of which is 13. *See* Sport X *and* XI.

IV. The ſum and the difference of any two numbers, the ſquares of which differ by a ſquare number, are each of them, either a ſquare number, or the half of one; thus, take the number 6 and 10, their ſquares 36 and 100 differ by the ſquare number

number 64; their ſum is 16, and their difference 4, each of which is a ſquare number: then take 8 and 10 for the two numbers, their ſquares 64 and 100, differ by the ſquare number 36; and the ſum 18, and the difference 2, are the halfs of the two ſquare numbers 36 and 4.—

To find two numbers, the ſum and difference of which, are, each of them, a ſquare number, In which caſe, the ſquares of theſe two numbers will likewiſe differ by a ſquare number; pitch upon any two numbers, as 2, and 3, the product of their multiplication is 6, their ſquares are 4 and 9; 13 the ſum of the two ſquares, and 12 the double of the product of their multiplications, are the numbers we look for; for their ſum 25, and their difference 1 are both ſquare numbers; and further, their ſquares 169 and 144 differ by the ſquare number 25.

To find two numbers, the ſum and difference of which are each of them the half or the double of a ſquare number; in which caſe their ſquares will likewiſe differ by a ſquare number; take any two numbers, as 2 and 3, the ſquares of which are 4 and 9; 13 the ſum of theſe two ſquares, and 5 the difference, are the two numbers demanded; for their ſum 18 and their difference 8, are the halfs of the two ſquare numbers 36 and 16, and the doubles of the two ſquares numbers 9 and 4; and farther, their ſquares 169 and 25, differ by the ſquare number 144 the root of which is 12.

V. Every ſquare number ends either with two cyphers, or with one of the 5 figures 1, 4, 5, 6, 9, which ſerves for a rule, *to diſtinguiſh when a number propoſed is not ſquare, viz.* when it does not end as above; nay, if it does end with two noughts, and theſe are not preceded by any of the

foregoing

foregoing five figures, we may reſt aſſured it is not ſquare.

VI. Every ſquare fraction, that is, every fraction that has its ſquare root, is ſuch, that the product of the multiplication of the numerator by the denominator is ſquare. Thus *we know a fraction is not ſquare*, when that does not happen. Take the fraction $\frac{28}{63}$, we know it to be ſquare, becauſe 1764, the product of 28, multiplied by 63, is a ſquare number, having 42 for its root; and ſo the ſquare root of the propoſed fraction is $\frac{42}{63}$, retaining the ſame denominator; or $\frac{28}{42}$, retaining the ſame numerator, for either of theſe is equivalent to $\frac{2}{3}$, for the ſquare root of the propoſed fraction $\frac{28}{63}$ or $\frac{4}{9}$.

VII. Any cubical fraction, *i. e.* any that has its cube-root, is ſuch, that if you multiply the numerator by the ſquare of the denominator, or the denominator by the ſquare of the numerator, the product has its cube-root; and it is by this rule that *we know when a fraction is a cube fraction, ſuch is* $\frac{24}{375}$ for 3375000, and 216000, the two products of the two ways of multiplication juſt mentioned, have 150 and 60 for their cube roots, and ſo the cube root of the fraction $\frac{24}{375}$ is $\frac{150}{375}$ retaining the ſame denominator, or $\frac{24}{60}$ retaining the ſame numerator, for each of theſe fractions is equal to $\frac{2}{5}$ as the cube root of the propoſed fraction $\frac{24}{375}$.

VIII. Though it is not poſſible to find two homogeneous powers, the ſum and difference of which are each of them a power of the ſame degree, that is, ſquare numbers if the two firſt are ſquares, and cube-numbers if they are cubical, &c. yet it is poſſible, and very eaſy, *to find two triangular numbers, the ſum and difference of which, are each of them a triangular number.*

Thus 15 and 21 are two triangular numbers, the ſides of which are five and 6, and their ſum 36;

and the difference 6 are likewise triangular numbers, having 8 and 3 for their sides. Again 780 and 990 are triangular numbers, the sides of which are 39 and 44, and their sum 1770 and the difference 210 are likewise triangular numbers, having 59 and 20 for their sides. Once more, 1747515 and 2185095 are triangular numbers, having 1869 and 2090 for their sides; and their sum 3932610 and the difference 437580 are likewise triangular numbers, the sides of which are 2804 and 935.

By a *Triangular Number* we understand the sum of the natural numbers, 1, 2, 3, 4, 5, 6, beginning with unit, and rising to what number you will, the last and the greatest of which is called the side. Thus we know that 10 is a triangular number, the side of which is four, by reason that it is equal to the sum of the first four natural numbers, 1, 2, 3, 4, the last and greatest of which is 4. It was called *Triangular*, because you may dispose 10 points in the form of an equilateral triangle, each side of which contains 4, and hence it was that 4 got the name of the side of the triangular number 10.

To know if a number proposed is triangular, you must multiply it by 8, and add 1 to the product, for if the sum be square, the proposed number is triangular. Thus we know that 10 is triangular, because 81 (the sum of its multiplication by 8, with with the addition of 1) is a square number, having 9 for its root.

IX The difference of two homogeneous powers, as of two square numbers, of two cube numbers, *&c.* is divisible by the difference of their sides. Accordingly we find that 21, the difference of the two square numbers 25 and 4, the sides of which are 5 and 2, is divisible by 3, the difference of the sides or roots, the quotient 7 being always equal

equal to the ſum of the ſame ſides or roots; and that 117, the difference of the cubes 125 and 8, the roots of which are 5 and 2, is diviſible by 3 the difference of the roots, the quotient 39 being equal to the product of the ſaid roots multiplied one into another, *viz.* 10, added to 29 the ſum of their ſquares 25 and 4.

X. The difference of the two homogeneal powers, the common exponent of which is an even number, is diviſible by the ſum of their roots. Thus, 21 the difference of the two ſquare numbers 25 and 4, the roots of which are 5 and 2, is diviſible by 7, the ſum of the ſaid roots, the quotient 3 being equal to the difference of the roots, and 609 the difference of the bi-quadrats 625 and 16, the roots of which are 5 and 2, is diviſible by 7, the ſum of the roots, the quotient 87 being equal to the product ariſing from 3, the difference of the roots, multiplied with 29 the ſum of their ſquares 25 and 4.

XI. The ſum of two homogeneal powers, the common exponent of which is an odd number, is diviſible by the ſum of their roots. Thus we know that 133 the ſum of the two cubes 125 and 8, the roots of which are 5 and 2, is diviſible by 7 the ſum of theſe roots, the quotient 19 being equal to the exceſs of the ſum of the ſquares of the roots (29) above the product of the roots (10). And that 3157 the ſum of the two ſur-ſolids 3125 and 32, the roots of which are 5 and 2, is diviſible by 7 the ſum of the roots; the quotient 451 being equal to the exceſs of 741 the ſum of the bi-quadrat powers of the roots 5 and 2 (625, 16) and of the ſquare of the product of the roots (100) its exceſs, I ſay, above 290 the product of the ſum of the ſquares of the ſame roots (29) multiplied by 10 the product of the roots themſelves.

XII. All the powers of the natural numbers 1, 2, 3, 4, 5, 6, *&c.* have as many differences as their exponents contain units, the laſt differences being always equal among themſelves in each power, that is, the ſecond differences, or the differences of the differences, in the ſquares 1, 4, 9, 16, 25, 36. *&c.* for theſe ſecond differences make 2, the firſt being the uneven numbers 3, 5, 7, 9, 11, *&c.* The third differences, or the differences of the differences of the firſt differences in the cubes 1, 8, 27, 64, 125, 216. *&c.* for theſe third differences make 6, the firſt being 7, 19, 37, 61, 91, *&c.* and the ſecond differences, *i, e,* the differences of theſe differences being 12, 18, 24, 30, *&c.* which riſe by 6 for the third difference, and ſo of the reſt.

The ſame thing happens to Pollygon numbers formed by the continual addition of numbers in continual arithmetical progreſſion, which are called Gnomons, and of which the firſt is always an unite, which is virtually any Polygon number. The ſame is the caſe with Pyramidal numbers, which are formed by the continual addition of Polygon numbers conſidered as Gnomons, the firſt of which is always unit: and in like manner with the Pyramido-Pyramidal numbers, which are produced by the continual addition of Pyramidal numbers, conſidered as Gnomons, the firſt of which is always unity.

When the Gnomons riſe, or exceed one another by one, as 1, 2, 3, 4, 5, 6, *&c,* the Polygon numbers 1, 3, 6, 10, 15, 21, *&c.* which are formed from them are called Triangular, the property of which is ſuch, that each of them being multiplied by 8, and the product inlarged by unity, the ſum is a ſquare number, as we intimated above. And farther, 9 the ſum of the ſecond and the third, omitting

omitting the firſt, is a ſquare number, and 36 the ſum of the fifth and the ſixth, omitting the fourth, is likewiſe ſquare, and ſo on.

When the Gnomons riſe, or exceed one another by two unites, as the odd numbers 1, 3, 5, 7, 9, 11, *&c.* the Polygon numbers formed from them, 1, 4, 9, 16, 25, 36, *&c.* are ſquare numbers; and when the Gnomons increaſe by three unites, as 1, 4, 7, 10, 13, 16, *&c.* the numbers formed from them, 1, 5, 12, 22, 35, 51, *&c.* are called Pentagons, and have this peculiar quality that each of them being multiplied by 24, and 1 added to the product, the ſum is a ſquare number, by which rule *we know when a propoſed number is Pentagon*, and ſo of the others.

To find the ſum of as many triangular numbers as you will, commencing from unit, of theſe eight for inſtance, 1, 3, 6, 10, 15, 21, 28, 36, multiply the given number 8 by the next follower 9, and the product 72, by the next after that 10, and divide the ſecond product 720 by 6, the quotient gives you 120 the ſum demanded.

The ſum of all theſe infinite fractions $\frac{1}{3}, \frac{1}{6}, \frac{1}{10}, \frac{1}{15}, \frac{1}{21}$, *&c.* the common numerator of which is 1, and the denominators of which are triangular numbers, their ſum, I ſay, is juſt 1.

To find the ſum of as many ſquare numbers from an unit as you will, of theſe eight, for example, 1, 4, 9, 16, 25, 36, 49, 64, take 36, the laſt of as many triangular numbers, *viz.* 1, 3, 6, 10, 15, 21, 28, 36, from 240 the double of this ſum 120, and the remainder 204 is the ſum you want.

XIII. The cubes, 1, 8, 27, 64, 125, 216, *&c.* of the natural numbers, 1, 2, 3, 4, 5, 6, *&c.* are ſuch, that the firſt 1 is a ſquare number, the root of which 1 is the firſt triangular number; the ſum of the two firſt, 1 and 8, *viz.* 9, is a ſquare number,

 the

the root of which 3 is the second triangular numbers 36 the ſum of the three firſt, 1, 8, 27 is a ſquare number, the root of which 6 is the third triangular number, and ſo on. And therefore if you want *to find the ſum of any number of cubic numbers from an unit*, of theſe ſix for example, 1, 8, 27, 64, 125, 216,) the ſquare of the ſixth triangular number 21, (441) is the ſum you deſire.

XIV. Among whole numbers, there is only 2 that being added to itſelf, makes as much as when multiplied by itſelf, *viz.* 4, for all other numbers make more by multiplication than by addition.

Though we cannot find two whole numbers, the ſum of which is equal to the product of their multiplication, yet we can eaſily *find two fractional numbers, and even in a given Ratio, the ſum of which is equal to their product, viz,* by dividing the ſum of the two terms of the given Ratio by each of the two terms; thus, if you give them the Ratio of the two numbers, 2, 3, divide their ſum 5 ſeparately by 2 and by 3, and you will have the two numbers $2\frac{1}{2}$, $1\frac{2}{3}$, which make as much when added together as when multiplied together, *viz.* $4\frac{1}{6}$.

XV. Any number is the half of the ſum of two others equally remote, the one in the way of defect, and the other in exceſs. For example, 6 is the half of 12, the ſum of the two numbers equally remote, 5 and 7, or 4 and 8.

XVI. The number 37 has this property, that being multiplied by any of theſe numbers, 3, 6, 9, 12, 15, 18, 21, 24, 27, which are in continual arithmetical progreſſion, all the products are compoſed of one figure thrice repeated.

37	37	37	37	37	37	37	37	37
3	6	9	12	15	18	21	24	27
111	222	333	444	555	666	777	888	999

XVII.

XVII. The two numbers 5 and 6 are called Spherical, because their powers terminate in these very numbers. The powers of 5, *viz.* 25. 125, 625, *&c.* terminate in 5; and in like manner the powers of 6, *viz.* 36, 216, 1296, *&c.* end with 6.

5 has that peculiar quality, that when multiplied by an odd number (as 7) its product terminates in 5, (as 35) and when multiplied by an even number (as 8) its product ends in a cypher, (as 40)

The other number, 6, has likewise this singular quality, that it is the first of the numbers which we call *perfect*, as being equal to the sum of their aliquot parts, for 6 is equal to the sum of its aliquot parts 1, 2, 3; 28 is likewise a *perfect* number, in regard it is equal to the sum of its aliqout parts 1, 2, 4, 7, 14: and one may find an infinity of other perfect number, as 496, which is equal to the sum of its aliquot parts 1, 2, 4, 8, 16, 31, 62, 124, 248.

To find all the perfect numbers in order, make use of the powers of 2, *viz.* 2, 4, 8, 16, 32, *&c*, and see which of these powers, when an unit is taken from them, makes a prime number, and you will find in 4, 8, 32, *&c.* that if you substract 1 from each of them, the remainders 3, 7, 31, *&c.* are prime numbers, each of which ought to be multiplied by the half of the corresponding power, that is, 3 by 2, 7 by 4, 31 by 16, *&c.* in order to obtain the perfect numbers 6, 28, 496, *&c.*

2	4	8	16	32
	1	1		1
	3	7		31
	2	4		16
	6	28		496

To find all the Aliquot Parts, or all the divisors of a proposed number, of which an unit is always one. If the number be 8128 (for example) which

is likewiſe a perfect number, divide it by the leaſt number that offers, *viz.* 2, which is eaſily done, becauſe 8128 is an even number, ſo the quotient will be 4064, which ſet down over againſt 2 for your ſecond diviſor, which may ſtill be divided by the firſt diviſor 2, and ſo its ſquare 4 may likewiſe be a diviſor, which ſet down under 2, over againſt the ſecond quotient 2032 for another diviſor, which may ſtill be divided by the firſt diviſor 2, and therefore its cube 8 will likewiſe be a diviſor, which you are to write under the ſquare 4, and oppoſite to the third quotient 1016 for another diviſor: thus you go on, till you come to the laſt diviſor that cannot be divided by 2, *viz.* the ſixth quotient 127 which being a prime number, that is, a number that can be divided by nothing but an unit, gives us to know that we have traced all the diviſors of the number propoſed 8128, and here you ſee the ſum of the diviſors is equal to the number propoſed, and by conſequence it is a perfect number.

1	
2	4064
4	2032
8	1016
16	508
32	254
64	127
———	———
127	8001
	127
	———
	8128

By the ſame method did we find out all the diviſors of the other number 2096128, which is likewiſe perfect, for as you ſee it is equal to the ſum of its aliquot parts. You ſee likewiſe that the laſt quotient 2047 which anſwers to 1024 the tenth power of the firſt diviſor 2 is alſo a prime number, for if it could have been divided by any other number beyond 2, as by 3, it behoved us to have multiplied

1	
2	1048064
4	524032
8	262016
16	131008
32	65504
64	32752
128	16376
256	8188
512	4094
1024	2047
———	———
2047	2094081

tiplied all the power of the firſt diviſor 2 by this new diviſor 3, and to have divided the number propoſed and all the quotient by this new diviſor 3, in order to have their diviſors, as you will ſee in the following example.

XVIII. The number 120 is equal to the half of 240, the ſum of its aliquot parts 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60. The number 672 is likewiſe equal to the half of 1344 the ſum of its aliquot parts, as will appear by obſerving the method above preſcribed, which we ſhall not now repeat. We may find a great many other numbers that have the ſame quality; nay, ſome may be found to be the third, or any other part of the ſum of their aliquot parts, which we ſhall not now inſiſt upon.

XIX. The two numbers 220 and 284 are called *Amiable*, becauſe the firſt 220 is equal to the ſum of the aliquot parts of the latter, 1, 2, 4, 71, 142; and reciprocally the latter 284 is equal to the ſum of the aliquot parts of the former, 1, 2, 4, 5, 10, 11, 22, 44, 55, 110. Theſe aliquot parts are eaſily found by what we have ſaid before, eſpecially if we conſider that all numbers that end in 5 or in 0, are diviſible by 5.

To find all the Amiable Numbers in order, make uſe of the number 2, which is of ſuch a quality, that if you take 1 from its tripple 6, from its ſextuple 12, from the octodecuple of its ſquare, 72, the remainders are the three prime numbers 5, 11, and 71, of which 5 and 11 being multiplied together, and the product 55 being multiplied by 4 the double of the number 2, this ſecond product 220 will be the firſt of the two numbers we look for; and to find the other 284, we need only to multiply the third prime number 71, by 4, the ſame double of 2, that we uſed before.

To find two other amiable numbers, inftead of 2 we make ufe of two of its powers that poffeffes the fame quality, fuch as its cube 8; for you fubtract an unit from its tripple 24, from its fextuple 48, and from 1152 the octodecuple of its fquare 64, the remainders are the three prime numbers, *viz.* 23, 47, 1151, of which the two firft, 23, 47, ought to be multiplied together, and their product 1081 ought to be multiplied by 16 the double of the cube 8 in order to have 17296 for the firft of the two numbers demanded. And for the other amiable number, which is 18416, we muft multiply the third prime number 1151 by 16 the fame double of the cube 8.

If you ftill want other amiable numbers, inftead of 2, or its cube 8, make ufe of its fquare cube 64, for it has the fame quality, and will anfwer as above.

In regard, it is difficult to know whether a number is prime if it be a large number, we fhall at the end of this Sport fubjoin a table of all the prime numbers between 1 and a 10000.

XX. The fquares of the two numbers 31, 34, *viz.* 961, 1156, are fuch, that the firft 961, with its aliquot parts, 1, 31, makes a fum (993) equal to 1, 2, 4, 17, 34, 68, 289, 578, the aliquot parts of the fecond 1156.

XXI. The two numbers 26, 20, make each of them with their aliquot parts, the fame fum; the firft 26 with its aliquot parts, 1, 2, 13, makes 42, and the fecond (20) with its aliquot parts 1, 2, 4, 5, 10, makes likewife 42.

The fame is the cafe of 488 and 464, each of them with their aliquot parts making 930: of 11 and 6, each of them with their aliquot parts making 12; and in fine of 17 and 10, which with their aliquot parts make 18 a piece.

Nay,

Nay, we may find three numbers, each of which with its aliquot parts makes the ſame ſum, as 20, 26 and 41, as alſo 23, 14, 15, and 46, 51, 71.

We may find two ſquare numbers of the ſame quality, particularly 16 and 25 the ſquares of 4 and 5; which are the loweſt that can be, and by virtue of which we come at as many more as we will of the ſame quality, *viz.* by multiplying them by ſome odd ſquare number, that is not diviſible by five. For example, if we multiply each of them by the ſquare number 9, we obtain two other ſqare numbers 144, and 225, each of which with its aliquot parts makes juſt 403.

XXII. 81 the ſquare of 9, with its aliquot parts, 1, 3, 9, 27, makes a ſquare number (121) the root of which is 11, 400 the ſquare of 20, with its aliquot parts, make the ſquare of 31 (961.)

XXIII. 666 the ſum of theſe three triangular numbers 15, 21, 630, the ſides of which are 5, 6, 35, is likewiſe a triangular number, the ſides of which is 36. The ſame is the caſe of theſe three triangular numbers 210, 780, 1711, and likewiſe of theſe 666, 2628, 5886.

XXIV. 49 the ſquare of 7 has this quality that 8 the ſum of its aliquot parts, 1, 7, is the cube of 2, and 343 the cube of the ſame number 7, does with its aliquot parts 1, 7, 49, make the ſquare number 400, the root of which is 20. I do not here pretend to direct you how to find out others of the ſame quality, for unleſs you light upon them by chance, it is very difficult to trace them without Algebra, which I propoſe not to mention in this performance.

XXV. 9 the ſquare of 3 has this quality, that 4 the ſum of its aliquot parts 1, 3, is the ſquare of 2, 2401 the ſquare of 49 has the ſame quality,

for

for 400, the ſum of its aliquot parts 1, 7, 49, 343 is the ſquare of 20.

XXVI. The two numbers 99, 63, have this quality, that (57) the ſum of the aliquot parts of the firſt, 1, 3, 9, 11, 33, ſurpaſſes (41) the ſum of the aliquot parts of the ſecond, 1, 3, 7. 9, 21. by the ſquare number 16, the root of which is 4. The ſame is the condition of 325 and 175; for the ſum of the aliquot parts of the firſt exceeds that of the aliquot parts of the other, by the ſquare number 36.

XXVII. The ſum of two numbers that differ by unity, is equal to the difference of their ſquares, and the ſum of the ſquares of their triangular numbers is likewiſe a triangular number. Thus 5 and 6 make the ſum 11 equal to the difference of their ſquares 25, 36, and their triangular numbers 15, 21, are ſuch, that 666, the ſum of their ſquares 225, 441, is likewiſe a triangular number, the ſide of which is 36.

XXVIII. The two triangular numbers, 6, 10, of the two numbers 3, 4, the difference of which is likewiſe an unity, have this quality, that their ſum 16, and their difference 4, are ſquare numbers, having 4 and 2 for roots; and 136 the ſum of their ſquares (36, 100) is a triangular number, the ſide of which 16 is likewiſe a ſquare number, the root of which is at the ſame time a ſquare number, having 2 for its ſide or root.

The ſame is the quality of the two other triangular numbers, 36, 47 the ſides of which, 8, 9, differ only by unity, for their ſum 81, and their difference 9, are ſquare numbers, the roots of which are 9 and 3, and 3321 the ſum of their ſquares (1296, 2025) is a triangular number, the ſide of which is 81, and that has its ſquare root 9, which again is the ſquare of 3.

There

There are many other triangular numbers of this quality, that may be found out by ſubſtracting and adding any ſquare number to its ſquare, the halves of the remainder and of the ſum being the two triangular numbers demanded. For example, if you ſubſtract 8 the ſquare number 16 from and add it to its ſquare 256, half the remainder 240, and half the ſum 272, preſent us with 120, and 136, fot the two triangular numbers thought for, the ſides of which are 15, 16, the difference conſiſting ſtill in unity.

Theſe two triangular numbers thus found, have this farther quality, that the greateſt of their ſides is always a ſquare number, and the difference of their ſquares is likewiſe a ſquare number; and withal their ſum is a bi-quadrate, equal to the ſquare of their difference, and at the ſame time to the ſide of the triangular number that compoſes the ſum of their ſquares.

XXIX. The difference of the ſquares of the two numbers in a duplicate Ratio, is equal to the ſum of their cubes divided by the ſum of their two numbers, and that very ſum of their cubes is the third of a cube.

Accordingly, 4 and 8 being in a duplicate Ratio, the difference 48 of their ſquares, 16, 64, is equal to the quotient reſulting from the diviſion of 576 (the ſum of their cubes, 64, 512) by 12 the ſum of the two numbers, and the very ſum of their cubes 576 is the third part of the cube 1728, the root of which 12 is always equal to the ſum of the two numbers.

I ſhould never have done, if I pretended here to fetch in all the properties of numbers, which indeed are infinite, and upon that conſideration I ſhall now conclude this Sport with the table of the prime numbers that I promiſed above.

Table

Table of the Prime Numbers between 1 *and* 10000.

1								
2	149	337	547	757	997	1223	1471	1699
3	151	347	557	761	—	1229	1481	—
5	157	349	563	769	1009	1231	1483	1709
7	163	353	569	773	1013	1237	1487	1721
11	167	359	571	787	1019	1249	1489	1723
13	173	367	577	797	1021	1259	1493	1733
17	179	373	587	—	1031	1277	1499	1741
19	181	379	593	811	1033	1279	—	1747
23	191	383	599	821	1039	1283	1511	1753
29	193	389	—	823	1049	1289	1523	1759
31	197	397	601	827	1051	1291	1531	1777
37	199	—	607	829	1061	1297	1543	1783
41	—	401	613	839	1063	—	1549	1787
43	211	409	617	853	1069	1301	1553	1789
47	223	419	619	857	1087	1303	1559	—
53	227	421	631	859	1091	1307	1567	1801
59	229	431	641	863	1093	1319	1571	1811
61	233	433	643	877	1097	1321	1579	1823
67	239	439	647	881	—	1327	1583	1831
71	241	443	653	883	1103	1361	1597	1847
73	251	449	659	887	1109	1367	—	1861
79	257	457	661	—	1117	1373	1601	1867
83	263	461	673	907	1123	1381	1670	1871
89	269	463	677	911	1129	1399	1609	1873
97	271	467	683	919	1151	—	1613	1877
—	277	479	691	929	1153	1409	1619	1879
101	281	487	—	937	1163	1423	1621	1889
103	283	491	701	941	1171	1427	1627	—
107	29	499	709	947	1181	1429	1637	1901
109	—	—	719	953	1187	1433	1657	1907
113	307	503	727	967	1193	1439	1663	1913
127	311	509	733	971	—	1447	1667	1931
131	313	521	739	977	1201	1451	1669	1933
137	317	523	743	983	1213	1453	1693	1949
139	331	541	751	991	1217	1459	1697	1951

1973	2237	2477	2749	3037	3329	3607	3889
1979	2239	—	2753	3041	3331	3613	—
1987	2243	2503	2767	3049	3343	3617	3907
1993	2251	2521	2777	3061	3347	3623	3911
1997	2267	2531	2789	3067	3359	3631	3917
1999	2269	2539	2791	3079	3361	3637	3919
—	2274	2543	2797	3083	3371	3643	3923
2003	2281	2549	—	3089	3373	3659	3929
2011	2287	2551	2801	—	3389	3671	3931
2017	2293	2557	2803	3109	3391	3673	3943
2027	2297	2579	2819	3119	—	3677	3947
2029	—	2591	2833	3121	3407	3691	3967
2039	2309	2593	2837	3137	3413	3697	3989
2053	2311	—	2843	3163	3443	—	—
2063	2333	2609	2851	3167	3449	3701	4001
2069	2339	2617	2857	3169	3457	3709	4003
2081	2341	2621	2861	3181	3461	3719	4007
2083	2347	2633	2879	3187	3463	3727	4013
2087	2351	2647	2887	3191	3467	3733	4019
2089	2357	2657	2897	—	3469	3739	4021
2099	2371	2659	—	3203	3491	3761	4027
—	2377	2663	2903	3209	3499	3767	4049
2111	2381	2671	2909	3217	—	3769	4051
2113	2383	2677	2917	3221	3511	3779	4057
2129	2389	2683	2927	3229	3517	3793	4073
2131	2393	2687	2939	3251	3527	3797	5079
2137	2399	2689	2953	3253	3529	—	4091
2141	—	2693	2957	3257	3533	3803	4093
2143	2411	2699	2963	3259	3539	3821	4069
2153	2417	—	2969	3271	3541	3823	—
2161	2423	2707	2971	3299	3547	3833	4111
2179	2437	2711	2999	—	3557	3847	4127
—	2441	2713	—	3301	3559	3851	4129
2203	2447	2719	3001	3307	3571	3853	4133
2207	2459	2729	3011	3313	3581	3863	4139
2213	2467	2731	3019	3319	3583	3877	4153
2221	2473	2741	3023	3323	3593	3881	4157

4159	4463	4759	5039	5381	5651	5903	6229
4177	4481	4783	5051	5385	5653	5923	6247
—	4483	4787	5059	5393	5657	5927	7257
4201	4493	4789	5077	5399	5659	5939	6261
4211	—	4793	5081	—	5669	5953	6269
4217	4507	4799	5087	5407	5683	5981	6271
4219	4513	—	5099	5413	5689	5987	6277
4229	4517	4801	—	5417	5693	—	6287
4231	4519	4813	5101	5419	—	6007	6299
4241	4523	4817	5107	5431	5701	6011	—
4243	4547	4831	5113	5437	5711	6029	6301
4253	4549	4861	5119	5441	5717	6037	6307
4259	4561	4871	5147	5443	5737	6043	6313
4261	4567	4877	5153	5449	5741	6047	6323
4271	4583	4889	5167	5471	5743	6053	6329
4273	4591	—	5171	5477	5749	6067	6337
4283	4597	4903	5179	5479	5779	6073	6343
4289	—	4909	5189	5483	5783	6079	6353
4297	4603	4919	5197	—	5791	6089	6355
—	4621	4931	—	5501	—	6091	6359
4327	4637	4933	5209	5503	5801	—	6361
4337	4639	4937	5227	5507	5807	6101	6367
4339	4643	4943	5231	5519	5813	6113	6373
4349	4649	4951	5233	5521	5821	6121	6379
4357	4651	4957	5237	5527	7827	6131	6389
4363	4657	4967	5261	5531	5839	6133	6397
4373	4663	4969	5273	5557	5843	6143	—
4391	4673	4973	5279	5563	5849	6151	6421
4397	4679	4987	5281	5569	5851	6163	6427
—	4691	4993	5297	5573	5857	6173	6449
4409	—	4999	—	5581	5861	6197	6451
4421	4703	—	5303	5591	5867	6199	6469
4423	4721	5003	5309	—	5869	—	6473
4441	4723	5009	5323	5623	5879	6203	6481
4447	4729	5011	5333	5639	5881	6211	6491
4451	4733	5021	5347	5641	5897	6217	—
4457	4751	5023	5351	5647	—	6221	6421

6429	6827	7121	7477	7723	8069	8369	8689
6547	6829	7127	7481	7727	8081	8377	8693
6551	6833	7129	7487	7741	8087	8387	8699
6553	6841	7151	7489	7753	8089	8389	——
6563	6857	7159	7499	7757	8093	——	8707
6569	6863	7177	——	7759	——	8419	8713
6571	6869	7187	7507	7789	8101	8423	8719
6577	6871	7193	7517	7793	8111	8429	8731
6581	6883	——	7523	——	2117	8431	8737
6599	6899	7207	7529	7817	8123	8443	8741
——	——	7211	7537	7823	8147	8447	8747
6607	6907	7213	7541	7829	8161	8461	8753
6619	6911	7219	7547	7841	8167	8467	8761
6637	6917	7229	7549	7853	8171	——	8779
6653	6947	7237	7559	7867	8179	8501	8783
6659	6949	7253	7561	7873	8191	8513	——
6661	6959	7243	7573	7877	——	8521	8803
6673	6961	7247	7577	7879	8209	8527	8807
6679	6967	7283	7583	7883	8219	8537	8819
6689	6971	7297	7589	——	8221	8539	8821
6691	6977	——	7591	7901	8231	8543	8831
——	6983	7307	——	7907	8233	8563	8837
6701	6991	7309	7603	7917	8237	8573	8839
6703	6997	7321	7607	7927	8243	8581	8849
6709	——	7331	7621	7933	8263	8597	8861
6719	7001	7333	7639	7937	8269	8599	8863
6733	7013	7349	7643	7949	8273	——	8867
6737	7019	7351	7649	7951	8287	8609	8087
6761	7027	7369	7669	7963	8291	8623	8893
6763	7039	7393	7673	7993	8293	8627	——
6779	7043	——	7681	——	8297	8629	8923
6781	7057	7411	7687	8009	——	8641	8929
6791	7069	7417	7691	8011	8311	8647	8933
6793	7079	7433	7699	8017	8317	8663	8941
——	——	7451	——	8039	8329	8669	8951
6803	7103	7457	7703	8053	8353	8677	8963
6823	7109	7459	7717	8059	8363	8681	8969

8971	9109	9239	9377	9479	9629	9767	8971
8999	9127	9241	9393	9491	9631	9769	9883
——	9133	257	9397	9497	9643	9781	9887
9001	9137	9277	——	——	9649	9787	——
6007	9151	9281	9403	9511	9661	9791	9901
9011	9157	9283	9413	9521	9677	9793	9907
9013	9161	9293	9419	9533	9679	——	9923
9029	9173	——	9421	9539	9689	9803	9929
9041	9181	9311	9431	9547	9697	9811	9931
9043	9187	9319	6433	9551	——	9817	9941
9049	9199	9323	9437	9587	9719	9829	9949
9059	——	9337	9439	——	9721	9833	9967
9067	9203	9341	9461	9601	9733	9839	9973
9091	9209	9343	9463	9613	6739	9851	——
——	9221	9349	9467	9619	9743	9857	
9103	9227	9371	9473	9623	9749	9859	

SPORT XI.

Of Right-angled Triangles in numbers.

BY a Rectangular Triangle in numbers, we mean three unequal numbers, the greateſt of which is ſuch that its ſquare is equal to the ſquare of the other two, ſuch are 3, 4, 5, for 25 the ſquare of 5 the greateſt, which we call the Hypothenuſe, is equal to the ſum of 9 and 16, the ſquares of the other two numbers, 3, 4, which we call the ſides, taking one for the Baſe of the right-angled triangle, and the other for the Altitude or Height. Half the product of the Baſe, and the Altitude, is called the Area, and is always diviſible by 3. The reader will obſerve all along that by the product of two numbers, we underſtand the number ariſing from their mutual multiplication.

There is an infinite number of right-angled triangles of divers ſorts, both in whole and in broken or fractional numbers, but we generally conceive them in integers, among which the firſt and the leaſt

leaſt of all is that now mentioned, 3, 4, 5, which has an infinity of fine properties, but it would be tedious to enumerate them, and therefore I ſhall content myſelf with obſerving, that the ſum (216) of the cubes (27, 64, 125) of the two ſides 3, 5, and of the Hypothenuſe (5) is a cube, the root or ſide of which (6) is equal to its Area.

To find in numbers as many Right-angled Triangles as you will: Take any two numbers, for example 2 and 3, which we call Generating Numbers, multiply them the one by the other, and (12) the double of their product (6) is the ſide of a right-lined triangle, the other ſide being equal to (5) the difference of the ſquares (4, 9) of the generating numbers, 2, 3, and the Hypothenuſe being equal to (13) the ſum of the ſame ſquares, 4, 9. And thus you have this right-angled triangle 5, 12, 13, for 169, the ſquare of the Hypothenuſe 13 is equal to the ſum of 25, 144, the ſquares of the two ſides 5, 12.

The firſt right-angled triangle, having 1, 2, for its generating numbers is ſuch, that the difference of the two ſides 3, 4, is 1; and if you want to *find another of the ſame quality*, take 2 the greateſt of theſe generating numbers for the leaſt of the two in the triangle demanded; and in order to find the greateſt for this ſecond triangle, add 1, the leaſt of the firſt, to 4 the double of the greateſt of the firſt, and ſo you have 5 for your greateſt generating number of the ſecond right-angled triangle, which conſequently is 20, 21, 29, where the difference of the two ſides 20, 21, is again 1.

If you deſire a third right-angled triangle of the ſame quality, make uſe of the laſt 20, 21, 29, after the ſame manner as you did the firſt, taking its greateſt generating number for the leaſt of the third, and adding its leaſt to the double of the greateſt, for the greateſt of this your third triangle;

 and

and ſo obſerving the ſame method you may find a fourth, fifth, *&c.* as appears by this table.

Sides		Hypoth.	Generat. numb.	
3	4	5	1	2
20	21	29	2	5
119	120	169	5	12
696	697	1025	12	29
4059	4060	5741	29	70
23660.	23661	33461.	70.	169.

The firſt right-angled triangle 3, 4, 5, has likewiſe this quality, that the exceſs of the Hypothenuſe 5 above the ſide 4, is alſo 1, foraſmuch as the difference of the two generating numbers is 1, and for this reaſon you may *find an infinite number of other right-angled triangles of this quality*, if for their generating you take two that differ only by unity, as you ſee in this table,

Baſes.	Altitude.	Hypoth.	Generat. numb.	
3	4	5	1	2
5	12	13	2	3
7	24	25	3	4
9	40	41	4	5
11	60	61	5	6
14	84	85	6	7

Here you ſee the firſt differences of the baſes, 3, 5, 7, 9, *&c.* are equal, and the ſecond differences of the altitudes, 4, 12, 24, 40, *&c.* are likewiſe equal; and the ſame is the caſe of the Hypothenuſes, 5, 13, 25, *&c.*

Here the Baſes are odd numbers, and if you *would have them the ſquares of theſe odd numbers*, only take the altitudes and Hypothenuſes for the generating numbers of the triangles you propoſe, which by conſequence will run thus

Baſes.

Bases.	Heights,	Hypoth.	Gen. numb.	
9	40	41	4	5
25	312	113	12	13
49	1200	1201	24	25
81	3280	3281	40	41
121	7320	7321	60	61
169	14280.	14281.	84.	85.

If Instead of one side you would have the *Hypothenuse to be the square number*, then your generating numbers must be the sides of a right-angled triangle, as in the following scheme, where you see the Hypothenuse is the square of the greatest generating number, with the addition of 1.

Sides.		Hypoth.	Gen. uumb.	
7	24	25	3	4
119	120	169	5	12
336	527	625	7	24
720	1519	1681	9	40
1320	3479	3721	11	60
2184	6887	7225	13	84

The right-angled triangle, 21, 28, 35, has this quality, that the two sides 21, 28, are triangular numbers, the sides of which, 6 and 7, differ only by unity, and the square (1225) of the Hypothenuse (35) is likewise a triangular number, the side of which is 49.

The same is the quality of the triangle 820, 861, 1189, as also of the triangle 28441, 28680, 40391, and of others.

The following right-angled triangles, which may be continued *in infinitum*, are such that their bases and hypothenuses are triangular numbers, and their heights cubic numbers.

Bases,	Heights.	Hypoth.	Gen. numb.	
6	8	10	1	3
36	27	43	3	6
120	64	136	6	10
300	125	135	10	15
630	216	666	15	21
1176	343	1225	21	28

You may *find as many ſuch triangles as you will*, by adding and ſubtracting a ſquare number from its ſquare, for in the addition half the ſum is the hypothenuſe, and in ſubſtracting half the remainder is the baſe, the height being equal to the cube of the root of the firſt ſquare number: or, which is the ſame thing, by taking for the generating numbers the triangular numbers in order, as you ſee in the ſcheme before us, where the leaſt generating numbers of one right-angled triangle is the greateſt of the preceding triangle.

SPORT XII.

Of Arithmetical Progreſſion.

BY Arithmetical Progreſſion, we mean a ſeries of quantities called Terms, that riſe continually by an equal exceſs, as 1, 3, 5, 7, 9, 11, *&c.* where the exceſs is 2; or 1, 4, 7, 10, 13, 16, *&c.* where the exceſs is 3; or 2, 6, 10, 14, 18, 22, *&c.* where they riſe by four at a time. And ſo of the reſt.

The principal property of arithmetical progreſſion, is this. Take three continual terms, as 6, 10, 14, the ſum (20) of the two extremes (6, 14,) is equal to the double of the middle term (10.) Take four continual terms, as 6, 10, 14, 18, the ſum (24) of the two extremes (6, 18) is equal to that of the two middle terms, (10, 14) In fine, in a larger number of continual terms, ſix for inſtance, as 2,

6, 10, 14, 18, 22, the ſum (24) of the two extremes (2, 22) is equal to that of any two terms that lie at an equal diſtance from them, as 6, 18, and 10, 14. From whence it is eaſy to conclude, that when a multitude of progreſſive terms is an odd number, the ſum of the extremes, or of thoſe equally remote, is the double of the middle term, as in theſe five terms, 2, 6, 10, 14, 18; for the ſum (20) of the extremes 2, 10, 18, or of the two equally remote, 6, 14, is the double of the middle term, 10.

You may readily find ſuch numbers as have this quality, that the ſum of their ſquares make a ſquare number, or, which is the ſame thing, the ſides of a right-angled triangle in numbers; and that by virtue of this double arithmetical progreſſion, $1\frac{1}{3}$, $2\frac{2}{5}$, $3\frac{3}{7}$, $4\frac{4}{9}$, where the exceſs is 2 in fractions, and 1 in whole numbers; for if you reduce the integer with its fraction to a fraction only, as $1\frac{1}{3}$ to $\frac{4}{3}$, the numerator 4 and the denominator 3 will be the ſides of the right-angled triangle 3, 4, 5; and in like manner if you reduce $2\frac{2}{5}$, to $\frac{12}{5}$ (which is done by multiplying the whole number 2 by the denominator 5, and adding to the product 10 the numerator 2) the denominator 5 and the numerator 12, will be the ſides of the right-angled triangle 5, 12, 13; and ſo of the reſt. Here you may ſee any odd number may be one of the ſides of a right-angled triangle in whole numbers.

Inſtead of the double arithmetical progreſſion, you may make uſe of this, $1\frac{7}{8}$, $2\frac{11}{12}$, $3\frac{15}{16}$, $4\frac{19}{20}$, $5\frac{23}{24}$, &c. where the exceſs is 4 in fractions, and 1 in whole numbers, for if you reduce $1\frac{7}{8}$ to $\frac{15}{8}$, the denominator 8, and the numerator 15, will be the two ſides of the right-angled triangle 8, 15, 17; and in like manner if you reduce $2\frac{11}{12}$ to $\frac{35}{12}$, the denominator 12 and the numerator 35 will be the

two ſides of another triangle 12, 35, 37; and ſo on. Here you ſee any odd number may be one of the ſides of a right-angled triangle in whole numbers.

In an arithmetical progreſſion, the ſum of the terms is equal to the ſum of the two extremes, multiplied by half the number of all the terms. And for this reaſon, in order *to find the ſum of any number of terms in Arithmetical Progreſſion*, for example, the ſum of theſe eight, 3, 5, 7, 9, 11, 13, 15, 17, you muſt multiply the ſum (20) of the two extremes (3, 17) by the number of the multitude of the terms (8) for then half the product (80 the half of 160) is the ſum you inquire for.

If on the other hand you know the ſum of the terms, the firſt term itſelf, and the number or multitude of the terms, you may find out what the terms are, by tracing the exceſs in this manner. Suppoſe the given ſum of the terms to be 80, the number of them 8, and the firſt term given 3, divide (160) the double of the ſum given (80) by the number given (8) then ſubtract from 20 the quotient, 6 the double of the firſt term given 3, and at laſt divide the remainder 14 by the given number wanting 1, that is 7, and the quotient 2 is the exceſs you look for, which added to the firſt term gives you five for the ſecond, and added to the ſecond 7 for the third and ſo on.

If the ſum of the terms, their number, and the exceſs be given, we find out the firſt term, and by conſequence all the reſt after the manner of the third queſtion enſuing.

Queſtion I. *A gentleman bargains with a bricklayer to have a well ſunk upon theſe terms; he is to allow him three livres for the firſt toiſe (a toiſe is 6 foot) of depth, 5 for the ſecond, 7 for the third, and ſo on, riſing two livres every toiſe till the*

the well is twenty toises deep. Query, how much will be due to the bricklayer, when he has digged twenty toises deep?

To resolve this question, multiply the 2 livres augmentation money at every toise, by the number of the toises, bating 1, that is by 19, to the product 38 add 6 the double of 3 the number of livres promised for the first toise, then multiply the sum 44 by half the number of all the toises, *viz.* 10, and the product shews you 444 livres due to the bricklayer for sinking the well 20 toises deep.

Quest. II. *A gentleman travelled* 100 *leagues in eight days, and every day travelled equally farther than the preceding day. Now it being discovered that the first day he travelled two leagues, the question is how many leagues he travelled on each of the other days.*

To resolve this question, divide 200, the double of the leagues given 100, by 8 the number of days given, and from the quotient 25, substract 4, the double of 2 the given number of leagues that he travelled the first day. Divide the remainder 21 by 7, the given number of days wanting one, and the quotient 3 shews that he travelled every day three leagues more than the day before, from whence it is easy to conclude, that since he traveled 2 leagues the first day, he travelled 5 the second, 8 the third, and so on.

Quest. III. *A traveller went* 100 *leagues in* 8 *days, and every day* 3 *leagues more than the preceding day. 'Tis asked how many leagues he travelled a day?*

Divide 200, the double of the leagues given 100, by 8 the number of days given, and from the quotient 25 subtract 21, the product of 3 the number of the daily increase multiplied by 7 the given number

of days bating 1. The remainder being 4 half it, and that ſhews you he travelled 2 leagues the firſt day; from whence it is eaſy to gather that he travelled 5 the ſecond, 8 the third, and ſo on.

Queſt. IV. *A robber being purſued travelled 8 leagues a day; an Archer, who was the purſuer, made but 3 leagues the firſt day, 5 the ſecond, 7 the third, and ſo on, increaſing two leagues every day. The queſtion is in how many days the archer will come up with the robber, and how many leagues they will have travelled.*

To reſolve this and ſuch like queſtions, add 2 the number of the daily increaſe of leagues by the archer, to 16 the double of 8 the number of leagues made every day by the robber: from the ſum 18 ſubtract 6 the duplicate of 3 the number of leagues that the archer travelled the firſt day. The remainder 12, divide by 2 the number of the archer's daily increaſe; and the quotient 6 will ſhew you, that the archer will come up with the robber at the end of 6 days, and conſequently both of them muſt by that time have travelled 48 leagues, for 6 times 8 is 48, and the ſame is the ſum of theſe ſix terms of Arithmetical Progreſſion, 3, 5, 7, 9, 11, 13.

Queſt. V. *We will ſuppoſe, it is 100 leagues from Paris to Lions, and that two couriers ſet out at the ſame time, and took the ſame road; one to go from Paris to Lions, making every day 2 leagues more than the day before, and the other from Lions to Paris, travelling every day 3 leagues farther than the preceding day; and that they met exactly half way, the firſt at the end of 5 days, and the other at the end of 4 days. Query, how how many leagues theſe two couriers travelled each day?*

To find how many leagues the courier travelled every day that was 5 days upon the road before he met the other; ſubtract 5 the number of days from

25

25 the ſquare of it, and having multiplied the remainder 20 by 2 the number of the daily increaſe of leagues for this courier; ſubtract the ptoduct 40 from 100, the number of leagues between Paris and Lions; and divide the remainder 60 by 10 the double of 5 the number of days; and the quotient 6 will ſhew you, that the courier travelled 6 leagues the firſt day, and conſequently 8 the ſecond, 10 the third, 12 the fourth and 14 the fifth.

In like manner with reference to the other courier, that arrived halfway in 4 days, ſubtract 4 the number of days from 16 its own ſquare, and having multiplied the remainder 12 by 3 the number of his daily increaſe of leagues, ſubtract the product 36 from 100, the diſtance of leagues fiom Paris to Lions; and divide the remainder 64 by 8 the double of 4 the number of days, and the quotient 8 will ſhew you that this courier travelled 8 leagues the firſt day, and conſequently 11 the ſecond, 14 the third, and 17 the fourth.

Queſt. VI. *There is a hundred apples and one basket ranged in a ſtrait line at the diſtance of a pace one from another; the queſtion is, how many paces muſt he walk that pretends to gather the apples one after another, and ſo put them into the baſket, which is not to be moved from its place?*

It is certain, that for the firſt apple he muſt make 2 paces, one to go, and another to return; for the ſecond 4, two to go and two to return; for the third 6, three to go, and ſo on in this arithmetical progreſſion, 2, 4, 6, 8, 10, &c. of which the laſt and greateſt term will be 200, that is, double the number of apples. To 200 the laſt term, add 2 the firſt term, and multiply the ſum 202 by 50, which is half the number of apples, or the number of the multitude of the terms; and the product 10100 will be the ſum of all the terms, to the number of paces demanded.

SPORT

SPORT XIII.

Of Geometrical Progreſſion.

BY Geometrical Progreſſion we underſtand a ſeries of ſeveral quantities that grow or riſe continually through the multiplication of one and the ſame number, as 3, 6, 12, 24, 48, 96, *&c.* where each term is the double of the precedent term; or, as 2, 6, 18, 54, 192, 486, *&c.* where each term is the triple of its antecedent. And ſo of others.

The principal property of Geometrical Progreſſion, is, that in three terms continually proportional, as 3, 6, 12, the product 36 of the two extremes, 3, 12, is equal to the ſquare of the middle term 6: and that in four terms in continual proportion, as 3, 6, 12, 24, the product 72 of the two extremes 3, 24, is the ſame with the product of the two means 6, 12: and in fine, that in a greater number of terms in continual proportion, as in theſe ſix, 3, 6, 12, 24, 48, 96, the product 288 of the two extremes 3, 96, is the ſame with that of 12, 24, two equally remote from it. From hence it is eaſy to conclude that when the number of the terms is odd, this product is equal to the ſquare of the mean, as in theſe five terms, 3, 6, 12, 24, 48; for 144 the product of the two extremes 3, 48, or of the two equally remote, 6, 24, is the ſquare of the mean 12.

Thus you ſee that what arithmetical progreſſion has by addition, geometrical progreſſion has it by multiplication. But there is another conſiderable difference between theſe two progreſſions, conſiſting in this; that in arithmetical progreſſion the differences of the terms are equal, and in geometrical progreſſion they are always unequal, and keep up among themſelves the ſame geometrical progreſſion,

by

by continuing *in infinitum*, the differences of differences, without ever coming to equal differences. Accordingly we ſee in this geometrical progreſſion 2, 6, 18, 54, 162, 486, the differences of the terms make juſt ſuch another geometrical progreſſion 4, 12, 36, 108, 324, and in this laſt progreſſion the differences of the terms make again the like geometrical progreſſion, 8, 24, 72, 216, and ſo on.

In three proportional terms, ſuch as 2, 6, 18, the cube 216 of the mean 6, is equal to the ſolid product of the three terms multiplied together: and in four numbers in continued proportion, ſuch as 2, 6, 18, 54, the cube 216 of the ſecond 6, is equal to the ſolid product ariſing from the multiplication of 54, the fourth term, by the ſquare of the firſt 2; and in like manner 5832 the cube of the third term 18, is equal to the ſolid product of the firſt term 2 multiplied by 2916 the ſquare of the fourth 54.

From what has been ſaid it is eaſy *to find a geometrical mean proportional between two numbers given*, by multiplying the one number by the other, and extracting the ſquare root for the mean proportional: and it is equally eaſy *to find two means in continued geometrical proportion to two numbers given*, as 2 and 54; by multiplying the laſt 54 by the ſquare of the firſt, and extracting the cube root (6) of the product (216) for the firſt mean proportional, which multiplied by the ſecond number 54, makes 324, and 18 the ſquare root of that product is the ſecond mean proportional.

But *to find an arithmetical mean proportional to two numbers given*, take half the ſum of the two numbers for the mean required; as in 2, 8 given, 5 the half of 10 is the mean: and *to find two arithmetical means in continued proportion* as between 2 and 11, we ſubtract the leaſt number 2 from the greateſt

greateſt 11, and add 3 a third part of the remainder 9, to the leaſt number 2, which gives us 5 for the firſt mean; as the addition of 6, the double of that third part, to the ſame leaſt number 2, does 8 for the ſecond. Or, if you will, you may add 4, the double of the leaſt 2, to 11 the greateſt, and reciprocally 22, the double of 11 the greateſt, to 2 the leaſt, and the thirds of the two ſums make 5 and 8 for the two means demanded.

It is evident that all the powers of the ſame number, as 2, riſing in order, make a geometrical progreſſion, ſuch as this, where you ſee the exponents of the powers

(1)	(2)		(4)				(8)	
2,	4,	8,	16,	32,	64,	128,	256,	*&c.*
1	1		1				1	
——	——		——				——	
3	5		17				257	

(1) (2) (4) (8) are the terms of a geometrical proportion, *viz.* 2, 4, 16, 256, *&c.* and all the powers ate ſuch that if you add an unite to each of them, the ſums 3, 5, 17, 257, *&c.* are prime numbers: and ſo it is eaſy to find a prime number greater than any number given.

If you continue a geometrical progreſſion upon the decreaſe *in infinitum* as 6, 2, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, *&c.* the difference 4 of the two firſt terms 6, and 2 is to the firſt 6, as the ſame number 6 is to the ſum of all the infinite terms. And therefore, *to find the ſum of all the infinite terms of a decreaſing geometrical proportion*, as that above, you muſt divide 36 the ſquare of the firſt term 6, by 4 the difference of the two firſt terms, and the quotient 9 is the ſum you want. If you take from this quotient, 8 the ſum of the two firſt terms 6 and 2, the remainder 1 is the ſum of the infinite fractions continually proportional, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$, *&c.* And by the ſame means we are

are taught that the ſum of other infinite fractions in continued proportion, amounts likewiſe to 1. This rule gives the ſolution to the following queſtion: but before I propoſe it, I muſt acquaint you, that,

When we ſpeak of quantities in proportion, without ſpecifying, we always mean geometrical proportion. Here I muſt obſerve by the by, that taking an unit for numerator, and the natural numbers, 1, 2, 3, 4, 5, *&c.* for denominators, if you make the following ſeries of fractions, $\frac{1}{1}\,\frac{1}{2}\,\frac{1}{3}\,\frac{1}{4}\,\frac{1}{5}$, *&c.* which ſtill decreaſe, theſe three taken conſecutively from three to three at pleaſure, will be in harmonic proportion; that is, the firſt of the three will be to the third, as the difference of the two firſt is to the difference of the two laſt; as will better appear by reducing theſe fractions to the ſame denomination, or to integers, by multiplying them by the number 60, which is diviſible by all the denominators 2, 3, 4, 5, for inſtead of the five fractions you have the five whole numbers, 60, 30, 20, 15, 12; of which the three firſt 60, 30, 20, are fairly in harmonic proportion for the firſt 60 is to the third 20 which is its third part, as 30 the difference of the two firſt is to 10 the difference of the two laſt, which is likewiſe the third part of 30. By the ſame conſideration you will perceive that theſe three, 30, 20, 15 are in harmonic proproportion as well as the other three 20, 15, 12.

Queſt. *A great ſhip purſues a little one, ſteering the ſame way, at the diſtance of four leagues from it, and ſails twice as faſt as the ſmall ſhip. It is asked how far the great ſhip muſt ſail before it overtakes the leſſer.*

The diſtance of the two ſhips being 4, and their celerities being in a double Ratio, continue *in infinitum*, the double geometrical progreſſion, 4, 2,

1,

1, $\frac{1}{2}\frac{1}{4}\frac{1}{8}$, &c. the firſt and the greateſt term of which is 4; and find the ſum of all the infinite terms, by dividing 16 the ſquare of the firſt 4, by 2 the difference of the two firſt, and the quotient 8 directs that the great ſhip muſt make 8 leagues before ſhe can come up with the other.

SPORT XIV.

Of Magical Squares.

BY a Magical Square we underſtand a ſquare divided into ſeveral other ſmall equal ſquares, filled with terms of an arithmetical progreſſion, ſo tranſpoſed, that all of the ſame line or rank, whether londitudinal, tranſverſe, or diagonal, make the ſame ſum.

This is the ſquare here annexed, divided into 25 little boxes or ſquares in which the firſt 25 natural numbers are ſo tranſpoſed, that the ſum of each rank from above downward, or from the right to the left, or along the diagonals or diameters of the ſquares, is every way 65; which ſum 65 is in all an odd ſquare number, that is, it contains an odd ſquare number of places, *viz.* 25, and is equal to the product ariſing from 5, the root of the ſquare number 25; multiplied by 13 the middle term of the arithmetical progreſſion, 1, 2, 3, 4, &c.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

This

This sum is likewise found, by disposing the given terms of the arithmetical progression, according to their natural series, 1, 2, 3, 4, &c. in the square places, as you see here; for then the sum of each diagonal rank, that is the rank extending from one corner of the square to the other, is the sum demanded. This will likewise hold in even squares, or those which contain an even square number of boxes.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

In order to dispose magically in the boxes of an odd square: for instance, that of 25 boxes, having 5 for its side; to dispose, I say, as many given numbers in arithmetical progression, as 1, 2, 3, 4, 5, and so on till you come to the last, and greatest 25; write the first and the least immediately under the middle box, or that which possesses the center of the square; and moving diagonal wise to the right, write the second term 2 in the adjacent box, the lowermost of the next right hand rank. Here proceeding in the course of the diagonal from left to right you find no place for number 3, and so are to place it in the opposite or uppermost box of the rank into which it should have fallen. In like mannner, finding no place for 4, you are to place it in the opposite box of the rank that it falls to on the outside

Thus

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Thus you continue proceeding ſtill diagonal-wiſe to the right; but in regard 6 falls to a place that is already filled with 1, you muſt there take a retrogade diagonal courſe from the right to the left, and write 6 in the lowermoſt ſtation of the rank in which the foregoing term 5 was placed, and ſo there will remain an empty place between 5 and 6. This retrogade courſe muſt always be obſerved when you fall in with a ſtation already poſſeſſed. Continue to place the reſt in order, according to theſe rules till you come to the angle of the ſquare, where in this example ſtands 15: then, foraſmuch as you can no longer move diagonal-wiſe to the right, you muſt place the term 16 in the ſecond place (from the top) of the ſame rank; this done, the reſt may be placed as the former, without any difficulty.

There are ſeveral magical diſpoſitions both for odd and even ſquares; but theſe being difficult to underſtand, we reckon them improper for Arithmetical Recreations.

This ſquare was called Magical, from its being in great veneration among the Egyptians, and the

Pythagoreans

Pythagoreans their diſciples, who to add more efficacy and virtue to this ſquare, dedicated it to the ſeven planets divers ways, and engraved it upon a plate of the metal that ſympathized with the planet. The ſquare thus dedicated, was incloſed with a regular Polygon, inſcribed in a circle divided into as many equal parts as there were units in the ſide of the ſquare; with the names of the angels of the planet, and the ſigns of the zodiac written upon the void ſpaces between the Polygon and the circumference of the circle-circumſcribed. Through vain ſuperſtition they believed that ſuch a medal or Taliſman would befriend the perſon that carried it about him upon occaſion.

They attributed to Saturn the ſquare of 9 places or boxes, 3 being the ſide, and 15 the ſum of numbers in each row or column; to Jupiter the ſquare of 16 places, 4 being the ſide, and 34 the ſum of the numbers in each row; to Mars the ſquare with 25, 5 being the ſide, and 65 the ſum of numbers in each rank; to the Sun the ſquare with 36, 6 being the ſide, and 111 the ſum of each row; to Venus that of 49, 7 being the ſide, and 175 the ſum of numbers in each rank or column; to Mercury that of 64, 8 being the ſide, and 260 the ſum of each column; to the Moon the ſquare with 81 lodges, having 9 for its ſide, and 369 for the ſum of each column.

In fine, they attributed to imperfect matter, the ſquare with 4 diviſions, having 2 for the ſide; and to God the ſquare of only 1 lodge, the ſide of which is an unit, which multiplied by itſelf, undergoes no change. By virtue of this Sport, we are taught to reſolve the following Queſtion.

Queſtion. *To draw up in three ranks the nine firſt cards, from an ace to a nine, in ſuch a manner that all the points of each rank, taken either lengthwiſe or breadthwiſe, or diagonal wiſe, may make the ſame ſum.*

 Diſpoſe

4	9	2
3	5	7
8	1	6

Dispose the nine first natura numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, magically according to the directions laid down above, and as you see it done here, and place the cards according to their number, answerable to these figures.

8	256	2
4	16	64
128	1	32

Instead of an arithmetical progression, you may take a geometrical; for instance, this double progression, 1, 2, 4, 8, 16, 32, 64, 128, 256, *&c.* and placing them magically, as above, you will find the product of each rank will be equal, *viz.* 4096, which is just the cube of the middle term 16.

1260	840	630
504	420	360
315	280	252

Here we shall add by the by, one square more of 9 stations, in which the numbers of each rank taken any way, as above, are in harmonical proportion; and you may find as many other numbers of the same quality, as you will, if instead of the foregoing numbers you put letters, as you see it done underneath, where the literal magnitudes of each rank are harmonically proportional; and so by giving different value to the three undetermined letters, $a, b, c,$ you will have instead of literal quantities, numbers that will always preserve an harmonic proportion in each rank.

a	$\frac{2ac}{a+c}$	c
$\frac{2ab}{a+b}$	$\frac{2bc}{b+c}$	$\frac{2abc}{2ab+ab-ba}$
b	$\frac{2abc}{2ac+ab-bc}$	$\frac{abc}{ab+ac-bc}$

3	10	25	16	11
20	8	19	12	2
5	9	13	17	21
22	14	7	18	4
15	24	1	2	23

12.	25	6	19	3
5	11	24	8	17
16	4	13	22	10
9	18	2	15	21
23	7	20	1	14

1	20	23	16	5
4	18	9	12	22
15	7	13	19	11
24	14	17	8	2
21	6	3	10	25

11	24	17	10	3
4	12	25	18	6
7	5	13	21	19
2	8	1	14	22
23	16	9	2	15

11	22	9	20	3
2	14	25	8	16
19	5	13	21	7
10	18	1	12	24
23	6	17	4	15

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

SPORT XIV.

Of an Arithmetical Triangle.

BY an Arithmetical Trangle we mean the half of a ſquare like a magical ſquare, divided into ſeveral ſmall and equal ſtations or points, which contain the natural numbers 1, 2, 3, 4, *&c.* the triangular numbers 1, 3, 6, 10, *&c.* which are formed by the continual addition of the foregoing numbers; the Pyramidal numbers 1, 4, 10, 20, *&c.* formed by the contitual addition of the triangular; the Pyramido-Pyramidals 1, 5, 15, 35, *&c.* formed by the continual addition of the Pyramidal; and ſo on, as you ſee in the following cut.

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	
1	3	6	10	15	21	28		
1	4	10	20	35	56			
1	5	15	35	70				
1	6	21	56					
1	7	28						
1	8							
1								

Among

Among the different uſes of the arithmetical triangle, I ſhall only ſingle out thoſe relating to Cmobinations, Permutations, and the Rules of Game; the reſt being too ſpeculative for Arithmetical Recreations.

By Combinations we underſtand all the different choices that can be made of ſeveral things, the multitude of which is known, by taking them divers ways, one by one, and two, three and three, *&c.* without ever taking the ſame twice.

For Example, If you have four things expreſs'd by theſe four letters, *a*, *b*, *c*, *d*; all the different ways of joining two of them, as *ab*, *ac*, *ad*, *bc*, *bd*, *cd*; or three of them, as *abc*, *abd*, *acd*, *bcd*; theſe I ſay, are called Combinations. And from hence it is eaſy to apprehend, that when four things are propoſed, you may take them one by one four ways; two and two ſix ways; three and three four ways: and by fours only one way; ſo that 1 in 4 combines four times; 2 ſix times; 3 four times; and 4 only once.

To find in a greater number of different things, ſuch as ſeven; the divers combinations that may be made by taking them divers ways, whether by Addition or Multiplication: as, if you would know all the poſſible conjunctions of the ſeven planets, taking them two by two; that is to ſay, if you would know how often 2 combines in 7; add an unite to each of the two numbers given, 2, 7, and ſo you have 3, 8, which gives us to know, that in the third ſtation (reckoning from below upwards, or from above downwards) of the eighth diagonal of the arithmetical triangle, you'll have the number of combinations demanded, *viz.* 21.

Or elſe, the two numbers given being 2 and 7, add together all the numbers of the ſecond rank,

 till

till you come at the ſeventh diagonal, *viz.* 1, 2, 3, 4, 5, 6, and the ſum 21 is what you want.

When the number of things propoſed goes beyond 9, the triangle here delineated can't ſerve you; and therefore we ſhall give this general rule for any number whatſoever.

The two numbers given being 2 and 7, to know how often 2 the leaſt will combine in 7 the greateſt; make of them theſe two arithmetical progreſſions 2, 1, and 7, 6, which decreaſe by an unit, and ought to have but two terms, that is, as many as the leaſt number 2 has units. Then multiply together all the terms of each progreſſion, that is, 7 by 6, and 2 by 1; and divide the firſt product 42 by the ſecond 2, and the quotient 21 ſatisfies the demand.

By this, or the foregoing method, you'll diſcover that three combines in 7, 35 times; 4 likewiſe 35 times; 5, 22 times; and 6 only 7 times. Whence it follows, that the number of all the combinations poſſible of ſeven different things, taken 1 by one, by two's, by threes, by fours, by fives, by ſixes, and ſevens, amounts to 127, as appears by the addition of all the particular combinations, 7, 21, 35, 35, 21, 7, 1, which anſwer the numbers, 1, 2, 3, 4, 5, 6, 7. But you may find this total yet eaſier, by forming this double geomitrical progreſſion, 1, 2, 4, 8, 16, 32, 64, conſiſting of ſeven terms, anſwerable to the number of things combined, *viz.* 7; for the ſum of theſe terms, 127, is the number you look for; which may ſtill be found yet an eaſier way, *viz.* Subtract 1 from the propoſed number of things 7, and the remainder, 6, directs you to take the ſixth power (64) of the number 2; and the double of that power, bating an unit, 127, is the number deſired.

Before I diſmiſs this ſubject, I ſhall here ſet down two methods peculiar to two and 3, for finding out

how

how often theſe two numbers may be combined in any number of things. Suppoſe the number of things given is 7, you'll find how often 2 will combine in it, by ſubtracting the given number 7 from its ſquare 49; and taking (21) the half of the remainder, 42, for the number deſired. You'll find how often 3 may combine in 7, by adding 14, the double of 7 to 343, the cube of the ſame giving number 7, and ſubtracting from the ſum (357) the triple (147) of the ſquare (49) of the ſame number (7) for then the ſixth part (35) of the remaininder (210) ſhews you, that three will combine in 7 35 times.

There is another ſort of combinations, that may be called Permutation, in which we take the ſame thing twice; as if you would combine theſe three numbers by two's, 2, 5, 6, in order to know what different quantities they can produce, if you conſider the two firſt thus, 25, you'll call them twenty five; if thus, 52, you will call them fifty two; in like manner, the firſt and third taken thus, 26, is a quite different quantity from the ſame two taken thus, 62; and ſo of all others. From whence it appears, that the multitude or number of permutations is the double of that of combinations.

Permutations are of very good uſe in making anagrams, and ſometimes give very lucky hints; as in the word *ROMA*, the letters of which being tranſpoſed make this other word *AMOR*; but it is a much luckier hit that we meet with in theſe two Latin verſes;

Signa te, ſigna, temere me tangis & angis,
Roma tibi ſubito motibus ibit amor.

the letters of which being read backwards, form the ſame verſes.

We likewiſe make uſe of Permutations in playing at dice, to know the number of chances that attend

the engaging to throw with two dice, 9 for inſtance; it being certain that the perſon who engages has four chances for it; for 9 may come up four ways, by *quatre cinque*, by *cinque quatre*, by *tres ſix*, and again by *ſix tres* (according as the firſt or ſecond dye happens to appear.)

To give the joynt combinations of ſeveral letters; for example theſe four *A M O R*, that is to find the number of their ſimple permutations, by tranſpoſing them all poſſible ways; make this arithmetical progreſſion, conſiſting of as many terms as there are letters to combine together, which in this example are four; ſo that the firſt term is always an unit, and the laſt denotes the number of letters; then multiply together all the terms, and the product 24 is the number of permutations or different changes that theſe four letters *A M O R* can undergo, as you ſee here;

A M O R	M A R O	O A M R	R O M A
A M R O	M A O R	O A R M	R O A M
A O M R	M O A R	O M A R	R M A O
A O R M	M O R A	O M R A	R M O A
A R M O	M R A O	O R A M	R A M O
A R O M	M R O A	O R M A	R A O M

By the ſame way do we find the number of Permutations of any other number of letters, *viz.* By making a progreſſion of as many natural numbers as there are letters to combine, and multiplying together all the terms of the progreſſion. Thus you'll find that five letters may be tranſpoſed 120 ways; ſix 720, and ſo on as in the following table, where you ſee the 23 letters of the alphabet may be combined 25852016738884976640000 ways,

1	1. A.
2	2. B.
3	6. C.
4	24. D.
5	120. E.
6	720. F.
7	5040. G.
8	40320. H.
9	362880. I.
10	3628800. K.
11	39916800. L.
12	479001600. M.
13	6227020800. N.
14	87178291200. O.
15	1307674368000. P.
16	20922789888000. Q.
17	355687428096000. R.
18	6402373705728000. S.
19	121645100408832000. T.
20	2432902008176640000. V.
21	51090942171709440000. X.
22	1124000727777607680000. Y.
23	25852016738884976640000. Z.
24	620448401733239439360000.
25	15511210043330985984000000.

This table is eaſily calculated; for having diſcovered that four letters, for example, may be combined or tranſpoſed 24 ways; for if you multiply 24, the number of combinations, by 5 the next number, you have 120 for the combinations of five letters; and that multiplied by the next number 6, makes 720 for the combinations of ſix letters; and ſo on through all the ſucceeding letters.

By Parti, in the way of gaming, we underſtand the juſt diſtribution or adjuſtment of what money out

out of the ſtakes belonging to ſeveral players, who play for it ſo many games, or a certain number of parts or ſets, in proportion to what every one has ground to hope from fortune, upon the ſets he wants to be up.

For example, If two gameſters have ſtaked down forty piſtols, which is then no longer their property only by way of retalliation, they have a right to what chance may bring them, upon the conditions ſtipulated at the firſt agreement; ſuppoſe they were to play for theſe 80 piſtols three ſets, that the firſt had gained one ſet, and the ſecond none; that is, the firſt wants two ſets to be out and the ſecond three; theſe ſuppoſitions being laid down, and the gameſters having a mind to draw their ſtakes, without ſtanding to their chances, the juſt Quota appertaining to each, is what is called Parti, and is found out by the arithmetical triangle, after this manner:

Since the ſuppoſition runs, that the firſt gameſter wants 2 ſets, and the other 3, and the ſum of the two numbers 2 and 3 is 5; we muſt turn to the fifth diagonal of the arithmetical triangle, and there take five the ſum of the two firſt numbers 1, 4, by reaſon of the two ſets that the firſt gameſter is ſhort; and 11 the ſum of the other three, 6, 4, 1, by reaſon of the three ſets that the ſecond gameſter is ſhort: and theſe two ſums 5 and 11 give the reciprocal Ratio of the two Parti's inquired for; ſo that the Parti or Quota of the one or firſt is to that of the ſecond, as 11 to 5.

But to adjuſt theſe Quota's, that is, to aſſign each gameſter his poſſitive ſhare of the 80 piſtoles at ſtake, this number 80 muſt be divided into two parts proportional to the two terms 11, 5; and this is done by multiplying 80 by the two ſums 11, 5,

ſeparately

ſeparately, and dividing each of the two products, (880, 400.) by 16, the ſum of the two terms 11, 5; by which means you have 55 for the number of piſtoles due to the firſt gameſter that gained a ſet; and 25 for the other that gained none.

In like manner, if the firſt wants but 1 ſet to be out, and the ſecond 2, we add together theſe two numbers 1, 2, and their ſum being 3, turn to the third diagonal of the arithmetical triangle, and there take the firſt number 1, and the ſum 3 of the two others 2, 1; from theſe two numbers 1, 3, we learn that the firſt his Quota is to that of the ſecond as 3 to 1; and ſince the ſum of theſe two terms is 4, the conſequence is, that the firſt gameſter ought to have $\frac{3}{4}$ of the 80 piſtoles ſtaked, and the ſecond only $\frac{1}{4}$, that is, the firſt 60 piſtoles, and the other 20.

Hence it appears, that when the game is at this paſs, the firſt may lay upon the ſquare 3 to 1: and this we can likewiſe make out without the arithmetical triangle, after the following manner.

Since the firſt wants one ſet to be out, and the ſecond two, we muſt conſider, that if they went on with the game, and the ſecond gained a ſet, then the two gameſters would have equal chances, and ſo their quota's or dividends would be equal, it being a conſtant and a general rule, that the one ſhare of the firſt is to that of the ſecond, as the chances of the one are to thoſe of the other. And ſo in this ſuppoſition, each of them has a title to an equal half of the money. 'Tis therefore certain, that if the firſt gains the ſet that is to be played, he ſweeps all; but if he loſes it, he has a title to an equal half: and therefore if they have a mind to draw without playing the ſet, the firſt ought to have half the money at ſtake, and the half of the remain-

ing

ing half, that is $\frac{3}{4}$ of the whole ; ſo that $\frac{1}{4}$ remains to the ſecond, for 'tis evident, that if a gameſter has a right to a certain ſum, in caſe he gains, and to a leſſer in caſe he loſes, he has a right to the half of thoſe two taken together, if the game is thrown up.

This firſt caſe directs us to the ſolution of the ſecond, which ſuppoſes the firſt to want one ſet to be out, and the ſecond three ; for if the firſt gains the ſet, he ſweeps all the 80 piſtoles; if he loſes, it turns to the firſt caſe, as above, that is, he has a right only to $\frac{3}{4}$; and therefore, if the ſtakes are drawn without playing that ſet, his right is half of theſe two ſums taken together, *i. e.* $\frac{7}{8}$ or 70 piſtols $\frac{1}{8}$ or 10 piſtoles remaining to the ſecond.

This leads us to a reſolution of a third caſe. Supposing the firſt to be too ſets ſhort, and the ſecond three; for if the firſt gains the next ſet, he has a right to $\frac{7}{8}$ of the money, by the ſecond caſe ; if he loſes it, ſo that the ſecond wants only two to be out, as well as he, the money is to be equally divided between them. Upon the whole, the game ſtands thus ; if the firſt wins, he claims $\frac{7}{8}$, if he loſes, he claims $\frac{1}{2}$; and therefore, if the game is thrown up without playing this ſet, he claims the half of theſe two ſums put together, *i. e.* $\frac{11}{16}$ or 55 piſtoles, leaving $\frac{5}{16}$ or 25 to the ſecond.

The ſecond caſe leads us likewiſe to the ſolution of a fourth caſe, in which the firſt is ſuppoſed to be one ſet ſhort of the whole, and the ſecond four ; for if the firſt gains a ſet, he carries the 80 piſtoles ; if he loſes it, ſo that the ſecond lacks only three to be out, he claims $\frac{7}{8}$ by the ſecond caſe. Now ſince, in caſe of winning, he takes 80 piſtoles and in caſe of loſing $\frac{7}{8}$ of them, his dividend, upon throwing up, is the half theſe two ſums put together

her, that is, $\frac{15}{16}$, or 75 piſtoles, and ſo he leaves [illegible], or 5 piſtoles for the ſecond.

The fourth and third caſes lead us after the ſame manner, to the ſolution of a fifth, which ſuppoſes hat the firſt gameſter is two ſets ſhort, and the econd four; for if the firſt gains a ſet, and ſo lacks ut one to be out, he claims $\frac{15}{16}$, by the fourth aſe; and if he loſes it, ſo that the ſeconds wants ut three, he claims $\frac{11}{16}$ by the third; and conſequently, in caſe of drawing, his due is the half of heſe two ſums put together, that is, $\frac{13}{16}$, or 65 iſtoles, $\frac{3}{16}$, or 15 piſtoles being left for the ſecond. And ſo of the other caſes.

All theſe, and an infinite number of other caſes hat may happen, are ſolvable without the arithmetical triangle, after a different and an eaſy maner, as follows;

Take the fifth caſe for inſtance, which ſuppoſes he firſt to be two ſets ſhort, and the ſecond four; n this ſuppoſition the two gameſters want between them ſix ſets to be out: Take 1 off the 6, and ſince the remainder is 5, ſuppoſe theſe five letters of the ſame form *a a a a a*, to favour the firſt gameſter; and theſe five *b b b b b*, to favour the ſecond; make combinations of theſe ten letters, as you ſee it here done; where of 32 combinations, the firſt 26 to the left, having at leaſt two *a*, are taken for the number of chances that can make the firſt to win; becauſe he lacks two ſets; and the remaining 6 to the right, or where there are at leaſt four *b*, are taken for the number of chances upon which the ſecond may win; becauſe he wants four to be out.

aaaaa	*aaabb*	*aabbb*	*abbbb*
aaaab	*aabba*	*abbba*	*bbbba*
aaaba	*abbaa*	*bbbaa*	*babbb*
aabaa	*bbaaa*	*ababb*	*bbabb*
abaaa	*aabab*	*abbab*	*bbbab*
baaaa	*abaab*	*ababb*	*bbbbb*
	baaab	*baabb*	
	baaba	*babba*	
	babaa	*bbaba*	
	ababa	*babab*	

Thus it is plain, that the firſt his due is to that of the ſecond as 26 to 6, or, as 13 to 3.

In like manner to ſolve the third caſe, which ſuppoſes the firſt to want two ſets to be up, and the ſecond three, ſo that they want five between them; take 1 from the ſaid ſum 5, and ſince the remainder is 4, ſuppoſe theſe ſimilar letters *a a a a* to be favourable to the firſt, and theſe four *b b b b* to the ſecond, and combine theſe eight letters together, as you ſee it here done; where, of the 16 combinations, the firſt 11 to the left having at leaſt two *a*'s, muſt repreſent the number of chances that the firſt has for game, two ſets being what he wants; and the remaining 5 to the right having at leaſt three *b*'s, muſt be taken for the number of chances that can make the ſecond up, he being three ſets ſhort. Thus the claim of the firſt is to that of the ſecond as 11 to 5, &c.

aaaa	*aabb*	*abbb*
aaab	*abba*	*bbba*
aaba	*bbaa*	*bbab*
abaa	*baab*	*babb*
baaa	*baba*	*bbbb*
	abab	

The ſame 16 combinations will ſerve for the ſolution of the fourth caſe, in which the firſt was ſuppoſed to be one ſet ſhort, and the ſecond four; ſo that five is the number of ſets wanted between them

hem, as in the third caſe. For among theſe 16 ou will find 15 that have at leaſt one *a*, (anſwerble to the one ſet that the firſt wants) for the chanes upon which the firſt will win; and only one hat has four *b*'s, the ſecond being four ſets ſhort, vhich ſhews there is but one chance that can ſave he ſecond. Thus the firſt ſhare is to that of the econd, as 15 to 1. And ſo of all other caſes.

To know, when two are at play, what advanage one has, that engages to throw 6, for examle, with one dye at a certain nnmber of throws, nd firſt of all, at the firſt throw; we muſt onſider that his caſe is 1 to five; for he has but ne chance to win, and 5 to loſe upon; and conequently if he lays upon one throw, he ought to ay but one to 5.

To engage to throw 6 with one dye at two hrows, is the ſame thing, as to throw two dyes at a time, one of which is to be a 6; and in that caſe, he who throws has but 11 chances to win upon, ſince he may throw the firſt 6, and the ſecond 1, 2, 3, 4, or 5; or the ſecond 6, and the firſt 1, 2, 3, 4, or 5; or elſe both dyes ſixes; whereas he has 25 to loſe upon, as you ſee here. Where it is eaſy to conclude, that he who offers to throw with one dye at two throws, ought to ſet but 11 to 25.

1. 1	2. 1	3. 1	4. 1	5. 1
1. 2	2. 2	3. 2	4. 2	5. 2
1. 3	2. 3	3. 3	4. 3	5. 3
1. 4	2. 4	3. 4	4. 4	5. 4
1. 5	2. 5	3. 5	4. 5	5. 5

When you lay upon 6 at two throws, take notice that 36, the ſum of all the chances, 11, 25, is the ſquare of the given number 6; and that 25, the number of chances againſt him who throws, is the ſquare of the ſame number, wanting 1, that is, 5. And therefore to find the number of chances that

favour

favour him who is to throw, you need only take 1 from 12, the double of the number given, and the remainder 11 is the number required; which being subtracted from 36, the square of the former number 6, leaves 25 the remainder, which will always be a square number, and denote the chances against him.

To lay upon 6 at three throws with one dye, is the same as to lay upon 6 at one throw with three dice; and in that case, he who throws has 91 favourable chances, and 125 against him, and so ought to set but 91 to 125; thus you see he is at a loss who lays upon the square for 6 at three throws of one dye.

Take notice that the sum 216 of all the chances 91, 125, is the cube of the given number 6, when you engage to throw 6 at three throws with one dye; and that 125, the number of the chances against you is the cube of the same number given, less 1, *i. e.* 5. And therefore, to find the number of chances that favour the person that throws, you need only to subtract 125, the cube of the given number 6, wanting 1 (*i. e.* 5) from 216 the cube of the same number given.

By the same method we find out what advantage he has who proffers to throw 6 with one dye at four throws; for if we subtract from the fourth power or biquadrate 1296 of the given number 6, if we subtract, I say, from that, 625 the biquadrate of the same number, less one, or of 5, the remainder shews us 671 favourable chances for him that throws; the biquadrate 625 being the number of the chances against him: So that he who lays upon 6 at four throws has the odds on his side.

But he has a much greater advantage upon 6 at five throws with one dye, as appears by subtracting 3125, the fifth power of 5 (the given number, bating

bating 1) from 7776, the fifth power of the given number 6; for the remainder 4651, is the number of the favourable chances, and 3125, the fifth power ſubtracted, is the number of thoſe againſt him who throws.

If you want to know what advantage he has, who offers with two or ſeveral dice, to throw at one throw a determined raffle; for example two *tres*; you muſt conſider, that with two dice he has but one chance to ſave him, and 35 to looſe upon, ſince two dice can combine 36 different ways, that is, their 6 faces may have 36 different poſtures, as you ſee by this ſcheme;

1	1	2	1	3	1	4	1	5	1	6	1
1	2	2	2	3	2	4	2	5	2	6	2
1	3	2	3	3	3	4	3	5	3	6	3
1	4	2	4	3	4	4	4	5	4	6	4
1	5	2	5	3	5	4	5	5	5	6	5
1	6	2	6	3	6	4	6	5	6	6	6

This number 36, is the ſquare of 6, the number of faces, there being but two dice; but if there were three, the cube of 6, 216, would be the number of combinations: and if there were four, the biquadrate of 6, 1296 would be the number. And ſo on.

From what has been ſaid it is evident, that in engaging a determined raffle at one throw with two dice, one ought to lay but 1 to 35; and by a parity of reaſon, that he ought to lay 3 to 213 upon a determined raffle or pair-royal with three dice; and 6 to 1290 with four; for of the 216 chances of three dice, there's only three that can favour him, ſince three things can combine by two's only 3 ways; and of 1296 chances of four dice, only 6 can favour the thrower, ſince four things combine by two's 6 ways.

 But

But if you want to know what odds he lies under who proffers to throw a raffle of one ſort or t'other at the firſt throw of two or more dice; you may find without difficulty, that he ought to ſet but 6 to 30, or 1 to 5 upon two dice, ſince of the 36 chances of two dice, there is only 6 that can make a raffle; and that upon three dice, his caſe is 18 to 198, or 1 to 11, ſince of the 216 chances, that three dice can fall upon only 18 can produce a raffle.

SPORT XV.

Several dice being thrown, to find the number of points that ariſe from them, after ſome operations.

SUPPOSE three dice thrown upon a table, which we ſhall call A, B, C, bid the perſon that threw them add together all the uppermoſt points, and likewiſe thoſe underneath of any two of the three: for inſtance, B and C, A being ſet apart, without altering its face. Then bid him throw again the ſame two dice, B and C, and make him add to the foregoing ſum all the points of the upper faces, and withal the lowermoſt points, or thoſe underneath of one of them, C for inſtance, B being ſet apart near A without changing its face, for giving a ſecond ſum. In fine, order him once more to throw the laſt dye C, and bid him add to the foregoing ſecond ſum the upper points, for a third ſum, which is thus to be diſcovered. After the third dye C is ſet by the other two, without changing its poſture, do you come up, and compute all the points upon the faces of the three dice, and add to their ſum as many 7's as there are dice, that is, in this example 21, and the ſum of theſe is what you look for; for when a dye is well made, 7 is rhe number of the points of the oppoſite faces.

To

To exemplify the matter: Suppose the first thrown of the three dice, A, B, C, brought up 1, 4, 5; setting the first 1 apart, we add to these three points 1, 4, 5, the points 3 and 2 that are found under or opposite to the upper points 4 and 5 of the other two dice; and this gives me the first sum 15. Now suppose again that the two last dice are thrown, and shew uppermost the two points 3 and 6, we set that with the three points apart, near the die that had 1 before, and add to the foregoing sum (15) these two points 3 and 6, and withal 1 the point that is found lowermost in the die that is still kept in service, and had 6 for its face at this throw; thus we have 25 for the second sum. We suppose at last, that this third and last die being thrown a third time, it comes up 6, which we add to the second sum 25, and so make the third sum 31. And this sum is to be found out by adding 21 to 10 the sum of the points 1, 3, 6, that appear upon the faces or uppermost sides of the three dice then set by.

SPORT XVI.

Two dice being thrown, to find the upper points of each die without seeing them.

MAKE any one throw two dice upon a table, and add 5 to the double of the upper points of one of them, and add to the sum multiplied by 5, the number of the uppermost points of the other or the second die; after that, having asked him the joint sum, throw out of it 25, the square of the number 5 that you gave to him, and the remainder will be a number consisting of two figures; the first of which to the left representing the tens, is the number of the upper points of the first dye, and

the ſecond figure to the right repreſenting units, is the number of the upper points of the ſecond die.

We will ſuppoſe that the number of the points of the firſt die that comes up is 2, and that of the ſecond 3; we add 5 to 4, the double of the points of the firſt, and multiply the ſum 9 by the ſame number 5, the product of which opperation is 45, to which we add 3, the number of the upper points of the ſecond die, and ſo make it 48, then we throw out of it 25, the ſquare of the ſame number 5, and the remainder is 23, the firſt figure of which 2 repreſents the number of points of the firſt die, and the ſecond 3 the number of points of the ſecond die.

Another way of anſwering this Sport is this; aſk him who threw the dice, what the points underneath make together, and how much the under points of one ſurpaſs thoſe of the other; and if this exceſs is, for example, 1, and the ſum of all the lower points is 9, add theſe two numbers 1 and 9, and ſubtract the ſum 10 from 14; then take 2, the half of the remainder 4, for the number of the upper points of one of the dice; and as for the other die, inſtead of adding the exceſs 1, to the ſum 9, ſubtract it out of 9, and take the remainder 8 out of 14, 6 is the remainder, the half of which, 3, is the number of the upper points of the ſecond die.

A third way is this: bid the perſon who threw the dice, add together the upper points, and tell you their ſum, which we here ſuppoſe to be 5; then give him orders to multiply the number of the upper points of one die by the number of upper points of the other die, and to acquaint you in like manner with their product, which we here ſuppoſe to be 6: now having this product 6, and the preceding ſum 5, ſquare 5, and from its ſquare 25 ſubtract 24, the quadruple of the product 6, and the remainder is 1: then take the ſquare root of the

the remainder, which in its caſe is 1, and by adding it to, and ſubſtracting it from the foregoing ſum 5, you have theſe two numbers 6, 4, the halfs of which, 3, 2, are the numbers of the upper points of each die.

SPORT XVII.

Upon the throw of three dice, to find the upper points of each die, without ſeeing them.

ORDER the perſon that has thrown the dice, to place them near one another in a ſtraight line, and aſk him the ſum of the lowermoſt points of the firſt and ſecond die, which we here ſuppoſe to be 9; then aſk him the ſum of the points underneath of the ſecond and third, which we here ſuppoſe to be 5; and at laſt the under points of the firſt and third, which we put 6. Now, having theſe numbers given you, 9, 5, 6, ſubtract the ſecond number 5 from 15, the ſum of the firſt and third, 9 and 6; and the remainder 10 from 14; ſo there remains 4, the half of which 2 is the number of the upper points of the firſt die. To find the number of the upper points of the ſecond, ſubtract the third number 6 from 14, the ſum of the two firſt 9 and 5; and the remainder 8 from 14 again; ſo you have a ſecond remainder 6, the half of which, 3, is the number demanded. At laſt for the third die, ſubtract the firſt number 9 from 11, the ſum of the ſecond and third 5, 6, and the remainder 2 from 14; ſo you have a ſecond remainder 12, the half of which, 6, is the number of the upper points of the third die.

SPORT XVIII.

To find a number thought of by another.

ORDER the perſon to take 1 from the number thought upon, and after doubling the

remainder, to take 1 from it, and to add to the laſt remainder, the number thought upon. Then aſk him what that ſum is, and after adding 3 to it, take the third part of it for the number thought of. For example, let 5 be the number, take 1 from it, there remains 4; then take 1 from 8, the double of that 4, and the remainder is 7, which becomes 12, by the addition of 5, the number thought of; and that 12, by the addition of 3, makes 15, the third part of which, 5, is the number thought of.

Another way is this: After taking 1 from the number thought of, let the remainder be tripled; then let him take 1 from that triple, and add to the remainder the number thought of. At laſt, aſk him the number ariſing from that addition, and if you add 4 to it, you will find the fourth part of the ſum to be the number thought of. Thus 5, bating 1, makes 4, that tripled makes 12, which loſing 1 ſinks to 11, and enlarged by the acceſſion of 5, comes to 16, which, by the addition of 4, is 20, and the fourth part of that, *viz.* 5, is the number thought of.

3d way. Add 1 to the number thought of, double the ſum, and add 1 more to it, and then add to the whole ſum the number thought of. Having learned the ſum total, take 3 from it, and the third part of the remainder is what you look for. Thus, 5 and 1 is 6, and the double of that, enlarged by 1 is 13, which by the addition of 5, comes to 18; take 3 from that, the remainder is 15, the third part of which, 5, is the number thought of.

4th. Or elſe after adding 1 to the number thought of, bid the perſon triple the ſame, and add firſt 1 to it, and then the number thought of. At laſt aſk the ſum of this laſt addition, and after robbing it of 4, take the fourth part of the remainder for the number

number thought of. Thus, 5 and 1 is 6, the triple of which and 1 is 19, which, with 5 is 24, and that bating 4 is 20, the fourth part of which, 5, answers the Sport.

5th. Take 1 from 5, the number thought of, double the remainder 4, from which, 8, take 1, and likewife the number thought of; after which, afk for the remainder 2, and add 3 to it, fo you have your number.

6th. Let the perfon that thinks add 1 to the five, the number thought of, and to the double of that, 12, 1 more, and fubtract from the fum, 13, the number thought of; then afk for the remainder 8, and taking 3 from it, what you leave behind, 5, is the number thought of.

7th. Bid the perfon that thinks take 1 from 5, the number thought of; and 1 from 12, the triple of the remainder; and then the double of the number thought of; 10, from 11, the laft remainder. This done, afk for the remaimder of the third fubtraction, *viz.* 1, and adding 4 to it, you will find fatisfaction.

8th. Add 1 to the number thought of 5, adding 1 more to the tripple of that you have, 19, from which take 10, the double of the number thought of; then afk for the remainder, 9, from which take 4, and fo you are right,

9th. Order the perfon to tripple the number thought of, (5) andout of the triple number (15) to caft away the half, if it were poffible; and fince in this example it is not, to add one to it fo as to make it 16; the half of which 8, muft be trippled, and that makes 24. The perfon that thinks having done this, afk him how many 9's are in the laft triple (24); he anfwers two; fo you are to take 2 for every 9, which in this example makes 4, and by reafon of the 1 you gave to make the 15 an even number,

 you

you are here to repay it by addition to the 4, and ſo you have 5, the number thought of. If there happen to be no 9 in the laſt triple, the number thought of is 1.

10*th*. Bid him add 1 to the number thought of (which makes 6); then ſubtract it from it, and ſo it leaves (4) a remainder; then bid him multiply the ſum (6) into the remainder (4) and tell you the product. To this product 24 add 1, and of the ſum 25 take the ſquare root 5.

11*th*. Bid the perſon that thinks add 1 to the number thought of (which we all along ſuppoſe to be 5) and multiply the ſum (6) by the number thought of (5), then let him ſubtract the number thought of (5) from the product (30) and tell you the remainder (25) the ſquare root of which 5 is the number thought of.

12*th*. After taking 1 from the number thought of, bid him multiply the remainder (4) by the number thought of (5) and add to the product (20) the ſame number thought of, and tell you the ſum 25, of which you are to extract the ſquare root 5.

13*th*. Bid him add 2 to the number thought of, and clap a cypher to the right of the ſum, which makes 70; and to that add 12, to the ſum of which addition (82) let him clap another cypher, ſo as to make it 820. From this decuple (820) let him ſubtract 320, and tell you the remainder 500, from which you are to cut off the two cyphers (each of which did ſtill decuple the number it was put to) and ſo you have the number thought of 5.

14*th*. Let him add 5 to the double of the number thought of: to the ſum 15 let him add a cypher on the right-hand to decuple it; then let him add 20 to the ſum (150) and to the laſt ſum (170) ſet another decupling cypher; at laſt let him ſubtract 700 from the laſt ſum of all (1700) and diſcover

to

o you the remainder 1000, which from you are to strike off two cyphers to the right, and take the half of the remainder (10) for the number thought of.

These two last methods are not very subtile; for the last number being known, it is an easy matter, by a retrogade view, to find out the other numbers, and by consequence the number thought of. And upon that consideration we shall here subjoin two other methods that are more mysterious.

15*th*. Bid the person that thinks add 1 to the triple of the number thought of, and triple the sum (16) again: to which last sum (48) bid him add the number thought of (5); then ask him the sum of all (53) and from that take off 3, and the right hand cypher from the remainder 50; which leaves you 5 to the left for the number thought of.

16*th*. Bid him take 1 from the triple of the number thought of (15) and multiply the remainder (14) by 3; and add to (42) the product, the number thought of (5); then ask the sum of the addition, 47, to which add 3, and cut off from the sum 50 the cypher, which must needs be on the right-hand, and so leaves to the left the number thought of.

From these two last methods we may draw this inference, that *if we add an unit to the triple of any number* (as to 18 the triple of 6) *and the same number* (6) *to the triple of the sum* (57) *the second sum* (63) *will always terminate with* 3.

Another inference is, that *if we subtract an unit from* (18) *the triple of any number* (6) *and add the same number* (6) *to the triple of the remainder* (51 the triple of 17) *the sum* (57) *will always end with the figure* 7.

The last inference is, that this double Sport is impossible, *viz. To find a number of such a quality, that*

that if you add to, or ſubtract from its triple, an unit, and add the ſame number to the triple of the ſum of the remainder, the laſt ſum will be a perfect ſquare number; for as we ſhewed in Sport 9 no number ending in 3 or 7 can be a true ſquare. See the following Sport.

SPORT XIX.

To find the number remaining after ſome operations, without asking any queſtions.

LET another think of a number at pleaſure; bid him add to the double of it an even number, ſuch as you have a mind to. For example 8; then bid him ſubtract from half the ſum the number thought on, and what remains is the half of the even number that you ordered him to add before; and ſo you may roundly tell him you are ſure the remainder is 4. Though the demonſtration of this is eaſy, yet thoſe who are not appriſed of the reaſon will be ſurpriſed at it. However that you may light exactly on the number thought of, conceal your knowledge of the remainder 4, and bid him ſubtract that remainder, whatever it is, from the number thought of, if ſo be it be larger; or elſe if the number be leſs, to ſubtract it from the remainder; and then aſk him for the remainder of the laſt ſubtraction; for if you add this remainder to the half of the even number you gave him (*i. e.* 4 the half of 8) when the number thought of is larger than that of the half of the even number; or if you ſubtract the remainder from the ſame half (4) when the number thought of is leſs than it, you will have the number thought of. To exemplify the matter, let 5 be the number thought of, and 8 added to its double 10, which makes 18; the half of that is 9; and 5, the number thought of, ſubtracted from 9 leaves 4, the half of the additional num-

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ber 8 ; and if you take this half 4 from the number thought of 5, there will remain 1, which being added to the ſame half 4 (the number thought of being greater than that half) gives 5, the number thought of. In like manner if to 10, the double of 5 the number thought of, you add 12, you will have 22, the half of which is 11 ; and from thence taking the number thought of 5, there remains 6, the half of the additional number 12 : and if from that half 6 you take the number thought of 5, (which in this example is leſs than the ſaid half) there will remain 1, which being taken from the ſame half, ſince the number thought of is leſs than that half (6) leaves 5 for the number thought of.

But an eaſier way to anſwer the ſport is this: Bid the perſon that thinks, take from the double of the number thought of, any even number you will that is leſs, for example 4; then let him take the half of the remainder from the number thought of, and what remains will be 2, the half of the firſt number ſubtracted 4; and therefore to find the number thought of, bid him add the number thought of to that half 2, and then aſk the ſum, 7, from which you are to take the ſame half, and ſo there will remain 5 for the number thought of.

But another, and yet eaſier way is this: Bid him add what number you will to the number thought of, and multiply the ſum by the number thought of; for if you make him ſubtract the ſquare of the number thought of from the product, and tell you the remainder, you have nothing to do but to divide that remainder by the number you gave him to add before ; for the quotient is the number thought of. Thus 4 added to 5 (the number thought of) makes 9, which being multipled by 5, makes 49 ; from which take 25, the ſquare of the number thought

thought of, and there remains 20, which being divided by 4, leaves 5 in the quotient.

Or elſe bid the perſon that thinks, take a certain leſſer number from the number thought of, and multiply the remainder by the ſame number thought of; for if you make him take the ſquare of the number thought of from the product, and tell you the remainder; by dividing that remainder by the number you ordered to be taken from the number thought of, you have the number thought of in the quotient.

But of all the ways for finding out a number thought of, the following is certainly the eaſieſt; make him take from the number thought of what number you pitch upon that is leſs than it, and ſet the remainder apart; then make him add the ſame number to the number thought upon, and the preceding remainder to the ſum, for a ſecond ſum; which he is to diſcover to you, and the half of that ſum is the number thought of. Thus 5 being thought of, and 3 taken from it, the remainder is 2; and the ſame number 3 adder to 5 makes 8, and that, with the preceding remainder, 10, the half of which, 5, is the number thought of.

SPORT XX.

To find the number thought of by another, without asking any queſtions.

BID the other perſon add to the number thought of, its half if it be even, or its greateſt half if it be odd; and to that ſum its half or greateſt half, according as it is even or odd, for a ſecond ſum, from which bid him ſubtract the double of the number thought of, and take the half of the remainder, or its leaſt half, if the remainder be odd; and thus he is to continue to take half after half, till he comes to an unit. In the mean time you are to obſerve

observe how many subdivisions he makes, retaining in your mind for the first division 2, for the second 4, for the third 8, and so on in a double proportion, remembring still to add 1 every time he took the least half; and that when he can make no subdivision, you are to retain only 8. By this means you have the number that he has halfed so often, and the quadruple of that number is the number thought of, if so be he was not obliged to take the greatest half at the beginning, which can only happen when the number thought of is evenly even, or divisible by 4; in other cases, if the greatest half was taken at the first division, you must subtract 3 from that quadruple; if the greatest half was taken only at the second division, you subtract but 2; and if he took the greatest half at each of the two divisions, you are to subtract 5 from the quadruple, and the remainder is the number thought of.

For example, let 4 be the number thought of, which by the addition of its half 2, becomes 6, and that, by the addition of its half 3, is 9: from which 8, the double of the number thought of, being subtracted, the remainder is 1, that admits of no division; and for this reason you retain only 1 in your mind, the quadruple of which, 4, is the number thought of.

Again; let 7 be the number thought of; this being odd, the greatest half of it, 4, added to it makes 11, which is odd again; and so the greatest half of 11 added to 11, makes 17, from which we take 14, the double of the number thought of, and so the remainder is 3, the least half of which is 1, that admits of no further division. Here there being but one subdivision, we retain 2, and to that add 1 for the least half taken, so we have 3, the quadruple of which is 12. But because the greatest

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moiety was taken both in the firſt and ſecond diviſion, we muſt ſubtract 5 from 12, and the remainder 7 is the number thought of.

SPORT XXI.

To find out two numbers thought of by any one.

HAVING bid the perſon that thinks add the two numbers thought of (for example, 3 and 5;) order him to multiply their ſum (8) by their difference (2) and to add to the product (16) the ſquare (9) of the leaſt of the two numbers (3) and tell you the ſum, 25, the ſquare root of which 5, is the greateſt of the two numbers thought of. Then for the leaſt, bid him ſubtract the firſt product (16) from the ſquare (25) of the greateſt number thought of (5) and tell you the remainder, 9, of which the ſquare root 3 is the leaſt number thought of.

An eaſier way of doing it is this: Bid him add to the ſum of the two put together (8) their difference (2) and tell you the laſt ſum, 10, for the half of it, 5, is the greateſt number thought of. And as for the leaſt, bid him ſubtract the difference of the two numbers thought of from their ſum, and aſk him the remainder, 6, the half of which, 3, is the number you look for.

This Sport may likewiſe be ſolved after the following manner: Bid him ſquare the ſum of the two numbers (*which is 64 in this example;*) then bid him add to the leaſt number thought of (3) the double (10) of the greateſt (5) and multiply the the ſum (13) by the leaſt (3) and ſubtract the product (39) from the foregoing ſquare (64) and diſcover the remainder 25, the ſquare root of which is the greateſt number thought of; and as for the leaſt, order dim to add to the greateſt (5) the double (6) of the leaſt (3), and multiply the ſum (11) by the

greateſt

greateſt (5) and ſubtract the product 55, from the foregoing ſquare (64) and tell you the remainder (9) the ſquare root of which is 3, the leaſt number thought of.

Another, and a very eaſy way, is this : bid him multiply the two numbers (5, 3,) together; and then multiply the ſum of the two numbers (8) by the number you want to find, whether the greater or leſſer, and ſubtract the product of the two numbers (15) from that product (which is 40, if you want the greater, and 24, if you look for the leſſer number) and tell you the remainder, 25, or 9, the ſquare roots of which ſatisfies the demand.

Or elſe, bid him firſt take the product of the two numbers (15), then multiply their difference (2) by the number enquired for (3 or 5) and add to that product the product of the two numbers (15) if you want the greateſt, or ſubtract that product from the product of the two numbers, if you look for the leaſt. Then he telling you the ſum, or the remainder, their ſquare roots are the numbers in queſtion.

When the leaſt of the two numbers does not exceed 9, it is eaſy to find them out after this manner: let one be added to the tripple of the greateſt, and the two numbers thought of to the tripple of that ſum, and the total ſum diſcovered; from which you are to take off 3, and then the right-hand figure is the leaſt, and the left-hand figure the greateſt number thought of. Thus 5 and 3 being thought of, 1 added to the tripple of 5, is 16, and the triple of that (48) added to 8, the ſum of the two numbers, makes 56, which loſing 3, is 53; 3 the right-hand figure being the leaſt, and 5 on the left the greateſt number thought of.

SPORT

SPORT XXII.

To find ſeveral numbers thought on by another.

IF the quantity of numbers thought of is odd, aſk for the ſums of the firſt and ſecond, of the ſecond and third, of the third and fourth, and ſo on till you have the ſum of the firſt and laſt; and having written all theſe ſums in order, ſo that the laſt ſum is that of the firſt and laſt; ſubtract all the ſums of the even places from all thoſe in the odd places; and the half of the remainder is the firſt number thought of, which being ſubtracted from the firſt ſum, leaves the ſecond number remaining, and that ſubtracted from the ſecond, leaves the third number remaining, and ſo on to the laſt. For example, ſuppoſe theſe five numbers thought of, 2, 4, 5, 7, 8, the ſums of the firſt and ſecond, of the ſecond and third; and ſo on to the ſum of the firſt and laſt, and 6, 9, 12, 15, 10: and 24 the ſum of the even places, 9 and 15, being taken from 28, the ſum of the odd places, there remains 4, the half of which, 2, is the firſt number thought of, and that being taken from the firſt number 6, leaves 4 for the ſecond number, and 4 taken from the ſecond, 9, leaves 5 for the third, and ſo on.

If the quantity of numbers thought upon is even, aſk for the ſums of the firſt and ſecond, of the ſecond and third, of the third and fourth, and ſo on to the ſum of the ſecond and laſt; write them all in order, ſo that the ſum of the ſecond and laſt may be the laſt in order; take all the ſums in the odd places (excepting the firſt) from thoſe in the even, and the half of the remainder is the ſecond number thought of, and that taken from the firſt ſum, leaves the firſt number, which taken from the third ſum, leaves for a remainder the third number, and ſo on. Thus 2, 4, 5, 7, 8, 9, being the num-

bers thought of, the ſums propoſed, as above, are 6, 9, 12, 15, 17, 13. Then take 29 the ſum of 12 and 17 the odd places (excepting the firſt) out of 37 the ſum of 9, 15, 13, the three even ſtations, and the remainder is 8, the half of which, 4, is the ſecond number thought of; and that taken from 6, the firſt ſum, leaves 2 the firſt number, as the ſame ſecond number 4, taken from the ſecond ſum 9, leaves 5 for the third number, which taken from the third ſum 12, leaves 7 for the fourth, and ſo on.

When each of the numbers thought of conſiſts only of one figure, they are eaſily found in the following manner: let the perſon add 1 to the double of the firſt number thought of, and multiply the ſum by 5, then add to the product the ſecond number thought of. If there is a third number, add 1 to the double of the preceding ſum, and after multiplying the whole by 5, add to the product the third number thought of. In like manner, if there is a fourth number, bid him add 1 to the double of the laſt preceding ſum, and after multiplying the whole by 5, add to the product the fourth number thought of, and ſo on, if there are more numbers. This dome, aſk the ſum ariſing from the addition of the laſt number thought of, and ſubtract from it 5 for two, 55 for three and 555 for four numbers thought of, and ſo on, if there are more; and then the firſt left-hand figure of the remainder is the firſt number thought of, the next (moving to the right) is the ſecond, the next to that the third, and ſo on till you come to the laſt right-hand figure, which is the laſt number thoughr of.

For example, let 3, 4, 6, 9, be the numbers thought of, and 1 added to 6, the double of the firſt 3, and the ſum 7 multiplied by 5, the product

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of which, 35, with the addition of the second number 4, is 39; then 1 being added to 78, the double of 39, and the sum 79 multiplied by 5, the product 395, with the addition of the third number 6, is 401; and the double of that, with the addition of an unit is 803, which multiplied by 5 is 4015, and with the addition of the fourth number, 9, 4024. Now, if from this sum 4024, we take 555, the remainder is 3469, the four figures of which are the four numbers thought of.

But there is a method for this purpose that is still easier, *viz.* Let 1 be subtracted from the double of the first number, and the remainder multiplied by 5, to the product of which multiplication, let the second number thought of be added, then, if there be more numbers than two, let him add 5 to the last sum for a second sum; let 1 be taken from the double of this second sum, and the remainder multiplied by 5, and the third number added to that product; this done, if there are no more numbers thought of (otherwise you must add 5, and go on again) ask for the last sum, add 5 to it, and the figures of the whole sum will represent the numbers thought of, as above.

For instance, let 3, 4, 6, 9, be thought of; take 1 from 6, the double of the first 3, multiply the remainder 5 by 5, add to the product 25, the second number 4; to the sum 29 add 5, which gives you 34 for a second sum, take 1 from 68, the double of this second sum, multiply the remainder 27 by 5, and to the product 335, add the third number 6, which makes 341; add 5 to this last sum, then it makes 346, the double of which, wanting 1, is 691, and that multiplied by 5, 3455, which, with the addition of the fourth number 9, is 3464. Now adding 5 to this sum, you have 3469, the four figures of which represent the four numbers thought of.

SPORT

SPORT XXIII.

A person has in one hand a certain even number of pistoles, and in the other an odd number; it is required to find out in which hand is the even or the odd number.

LET the number in the right-hand be multiplied by any even number you will, as 2, and the number in the left by such an uneven number as you pitch upon, as 3, then order the person to add together the two products, and take the half of their sum, and if he can take an exact half, so that the sum is even, you will know by that, that the number in the right-hand being multiplied by an even number is odd, and consequently that in the left multiplied by an odd number is even. But on the contrary, if he cannot take an exact half, the number in the right is even, and that in the left odd.

For example; suppose 9 pistoles in the right-hand, and 8 in the left; multiply 9 by 2, and 8 by 3; the sum of the two products 42 being an even number, shews that 9, the odd number, multiplied by the even 2, is in the right-hand, and consequently 8 the even in the left. This Sport directs us to the solution of the following question.

Question. *A man having a piece of gold in one hand and silver in the other, it is asked in what hand the gold or silver is in?*

Fix a certain value in an even number, as 8, on the gold, and an odd, as 5, upon the silver. Direct the person to multiply the number answering to the right-hand by any even number, as 2, and that in the left by a determined odd number, as 3, and ask him whether the joint sum of the products is even or odd; or bid him half it, and so you will learn whether it is even or odd, without asking. If this sum is odd, the gold is in the right-hand; if even, è contra.

SPORT XXIV.

To find two numbers, the Ratio and difference of which is given.

TO find two numbers, the firſt of which, for example, is the ſecond as 5 to 2, and the difference or exceſs 12: multiply the difference 12 by 2, the *leaſt* term of the given Ratio, and divide the product 24 by 3, the difference of the two terms 5, 2, and you will find the quotient 8, the leaſt of the two numbers looked for, and that added to the difference 12, *viz.* 20, the greateſt.

If you will, you may multiply the given difference by the *greateſt* term of the given Ratio, and after dividing the product by the difference of the two terms of the Ratio, you will find the quotient the great number, which, upon the ſubtraction of 12, leaves the leſſer remaining. Or you may take this way; multiply each of the two terms of the given Ratio, by the difference given, and divide each of the products by the difference of the two terms, and the quotients are the numbers demanded. This Sport furniſhes an eaſy ſolution to the following queſtion.

Queſtion. *If a man has as many pieces of money in one hand as in the other, how ſhall we know how much is in each hand?*

Bid him put two out of the left into the right-hand, which by that means will have 4 more than the left, and aſk for the Ratio of the number of pieces in the right to that in the left, which we ſhall here ſuppoſe to be as 5 to 3. Then multiply 4, the difference of the two hands, by 3, the leaſt term of the given Ratio, and divide the product 12, by 2, the difference of the two terms of the Ratio 5, 3: the quotient 6 is the number of pieces in the left, to which if you add the difference 4, you have 10 for the right. Theſe two put together make 16, and conſequently at firſt the man had 8 in each hand.

SPORT XXV.

Two perſons having agreed to take at pleaſure leſs numbers than a number propoſed, and to continue it alternately, till all the numbers make together a determined number greater than the number propoſed, it is required how to do it.

SUPPOSE the firſt is to make up 100, and both he and the ſecond are are at liberty to take alternately any number under 11; let the firſt take 11 from 100 as often as he can, and theſe numbers will remain, 1, 12, 23, 34, 45, 56, 67, 78, 89, which he is to keep in mind; and firſt take 1, for then let the ſecond take what number he will (under (11) he cannot hinder the firſt to come at the ſecond number 12, for if the ſecond takes 3, for example, which, with 1 makes 4, the firſt has nothing to do but to take 8, and ſo reach 12. After that, let the ſecond perſon take what number he will, he cannot hinder the firſt from coming at the third number 23; for if he takes 1, for inſtance, which with 12 is 13, the firſt takes 10 and ſo makes 23. In like manner, the firſt cannot be hindered to reach the fourth number 34, then the fifth 45, then 56, then 67, then 78, then 89, and at laſt 100.

As for the ſecond perſon, he can never touch at 100, if the firſt underſtands the way. Indeed if the firſt takes 2 at the beginning, his buſineſs is to take 10, and ſo clap in upon 12, with the ſame advantage the firſt had above. But if the firſt is acquainted with the artifice, he will be ſure to take 1, and ſo the ſecond can never make 12, nor 23, *&c.* nor, in fine 100.

If the firſt would be ſure to win, he muſt take care that the leſſer number propoſed does not meaſure the greater; for if it does, he has no infallible rule to go by. For example, if, inſtead of 11, 10

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were the number proposed; taking 10 continually from 100, you have these numbers, 10, 20, 30, 40, 50, 60, 70, 80, 90; now the first being obliged to pitch under 10, cannot hinder the other from making 10, and so 20, 30, *&c.* and in fine 100.

You need not be at the pains to make a continued subtraction of the lesser number from the greater, in order to know the numbers the first is to run upon; for if you divide the greater by the lesser, the remainder of the division is the first number you are to take. Thus divide 100 by 11, 1 is the remainder for the first number, add to that 11, it makes 12 for the second, and 12 with 11 makes 23 for the third, and so on to 100.

SPORT XXVI.

To divide a given number into two parts, the Ratio of which is equal to that of two numbers given.

SUPPOSE 60 is to be divided into two numbers, the least of which must be to the greater as 1 to 2: add together the two terms of the given Ratio 1, 2, and divide 60 by their sum 3: the quotient 20 is the least number wanted, and that subtracted from 60 leaves 40 the greater. Or, multiply the two terms 1, 2, seperately, by 60, and divide each of the products, 60, 120, by 3, the sum of the terms; and the two quotients, 20, 40, are the numbers you look for. This Sport gives an easy solution to the following question.

Question. *To divide the value of a crown into two different species or denominations, the number of which shall be equal.*

The solution being demanded in integers, it is impossible to solve this or the like Sport, unless the sum of the two terms of the Ratio of the different species proposed does exactly divide the crown when reduced to smaller money. Thus it is impossible to divide an English crown according to the tenour of the

the Queſtion, into ſhillings and pence; becauſe the Ratio of theſe ſpecies or denominations is 12, 1: and 13, the ſum of theſe two terms, does not exactly divide 60 pence, the value of the crown: but make the two ſpecies pence and farthings it will do; ſince 4, 1, the terms of their Ratio, make together 5, which exactly divides 240, the value of the crown in farthings; and the quotient 48, ſolves the Queſtion, that is, 48 pence, and 48 farthings, make a crown.

SPORT XXVII.

To find a number which being divided by given numbers ſeparately, leaves 1 the remainder of each diviſion; and when divided by another number given leaves no remainder.

TO find a number which leaves 1 remaining, when divided by 5 and by 7, and nothing when divided by 3: multiply into one another the two firſt numbers given, 5, 7; to their product 35, add 1, which makes 36, the number demanded. For if you divide 36 by 5 and by 7, the remainder is 1; and when you divide it by 3, there is as it happens, no remainder.

After finding this firſt and loweſt number of the propoſed quality 36, you may find an infinite quantity of greater numbers of the ſame quality, and that in the following manner. Add the firſt number found 36, to 105, the product of the three given nnmbers 5, 7, 3, and the ſum 141 is a ſecond number of the ſame quality, then add to 141 the product above-mentioned 105, and you have 246 for a third; which, with the addition of 105, makes 351 for a fourth number; and ſo on.

To find a number that divided ſeparately by 2, 3, 5, leaves 1 remaining, and no remainder when divided by 11: if you take 30, the product of the firſt three numbers 2, 3, 5, and add one to it, you

have the number 31, which divided by each of the three first numbers, 2, 3, 5, there should remain 11, and by 11, the fourth number, nothing: but so it is, that 31, when divided by 11, leaves 9 remaining, and therefore 31 is not the right number; but in order to find out the right number, take 30 the product of the three terms 2, 3, 5, and quadruple it, which makes 120, which with the addition of 1, is the number required 121, and that added to 1320, the product of the four numbers given, 2, 3, 5, 11, makes 1441 for a second number of the same quality: and so on, as above. In this case, 30, the product of 2, 3, 5, being divided by 11, left 8 remaining, and the quadruple of that 8, 32, being but 1 short of 33, the multiple or triple of 11, we quadrupled the 30, and added to the sum.

In like manner, to find a number that divided separately by 3, 5, 7, leaves 2 remaining, and no remainder when divided by 8: divide 105, the product of the three first numbers 3, 5, 7, by the fourth 8; and because there remains 1, multiply the product 105 by 6, that the product 630 divided by 8 may leave a remainder of 6, which is less than 8 by 2, and then adding 2 to the last product 630, you have 632 the number required, which added to the product of the four given numbers, makes a second number of the same quality; and that with the same addition, a third, and so on.

To find a number that divided separately by 3, 5, 7, leaves 2 remaining, and divided by 11 leaves no remainder: divide 105, the product of the first three numbers given, 3, 5, 7, by the fourth 11; and in regard there remains 6, the double of which, 12, surpasses the divisor 11 by 1; multiply the product 105 by 2, that 210 being divided by 11, there may remain 1; and since it is desired that 9 may be the remainder, which is less than the divisor 11

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by 2, multiply the laſt product 210 by 9, and then the product 1890 being divided by 11, the remainder will be 9; and therefore adding 2 to that laſt product, you will have a number 1892, which leaves no remainder, being divided by 11.

In like manner to find a number that being divided by 5, or 7, or 8, leaves 3 remaining, and nothing when divided by 11: multiply by 9, 280 the product of the firſt three numbers given, 5, 7, 8, and the product 2520 being divided by 11, there remains 1, upon which you may make the remainder 8, which is leſs than 11 by the given number 3, by multiplying the foregoing product 2520 by 8, which makes 20163, and conſequently that ſum, with the addition of 3, *viz.* 20163, is the number ſought for. This Sport directs us to ſolve the following Queſtion.

Queſtion. *To find how many piſtoles were in a purſe that a man has loſt, but remembers, that when he told them by two's, by threes, or by fives, there always remained an odd one, and when he counted them by ſevens, there remained none.*

Here we are to find a number that, when divided by either 2, or 3, or 5, ſtill leaves 1 remainder; and when divided by 7 leaves 0. Now there are ſeveral numbers of that quality, as appears from the foregoing Sport; and therefore to find the number that really was in the purſe, it behoves us to be directed by the bulk or weight of the purſe, in order to determine the real number.

Now to find the leaſt of all theſe numbers, let us firſt of all try for a number that is exactly diviſible by 2, by 3, and by 5, and likewiſe by 7 when 1 is added to it. If you multiply together the three firſt numbers given, 2, 3, 5, their product 30 will be diviſible by each of theſe three numbers; but when you have added 1 to it, the ſum 31 is not diviſible

divisible by the fourth number given, 7, for there remains 3, and since the product 30, when divided by 7 leaves 2, its double 60 will leave 4 upon the like division, and by the same consequence its triple 90 will leave 6 remaining. Now 6 wanting but one of 7, add that one to this triple number 90, and so 91 will be exactly divisible by 7, and consequently is the number sought for.

To find the next larger number that answers the Question, multiply together the four given numbers 2, 3, 5, 7, and to their product 210 add the first and least number found 91; the sum 301 is the second number sought for; and if you add to this second number the foregoing product 210, the sum 511 will be the third number that solves the Question; and so on *in infinitum*.

Thus, to resolve the Question, you may answer, that there might be in the purse 91 Louis d'Ors, or 301, or 511; and the bulk of the purse will serve to direct you which of the numbers was really in it.

SPORT XXVIII.

Of several numbers given to divide each into two parts, and to find two numbers of such a quality, that when the first part of each of the given numbers is multiplied by the first number given, and the second by the second, the sum of the two products is still the same.

SUPPOSE, for example, these three numbers given, 10, 25, 30, and the solution is required in entire numbers; take any two numbers for the two numbers sought for, provided their difference be 1, or such as may exactly divide the product under the greatest of these two numbers and the difference of any two of the three given numbers, and so, that the greatest of these two numbers multiplied

multiplied by the leaſt given number 10, may be greater than the leaſt of theſe two numbers multiplied by the greateſt given number 30; ſuch are 2 and 7.

The two numbers required, 2 and 7, being thus found; the firſt part of the firſt given number 10, may be taken at pleaſure, provided it is leſs than 10, and then the number ariſing from the ſubtraction of the leaſt found number 2, multiplied by the greateſt given number 30, from the greateſt found number 7, multiplied by the leaſt given number 10; and then the number ariſing from the diviſion of the remainder 10 by 5, the difference of the two numbers found 2, 7; that is, leſs than 2, which is 1, which being ſubtracted from the firſt given number 10, leaves the remainder 9 for the other part; and that being multiplied by the ſecond number found 7, and the firſt part 1 being multiplied by the firſt number found 2, the ſum of the two products 63 and 2 is 65.

To find the firſt part of the ſecond number given, 25, multiply 15, the difference of the firſt two numbers given, 10, 25, by the greateſt number found 7; and divide the product 105 by 5 the difference of the two numbers found 2, 7; then add the quotient 21 to 1, the firſt part found of the firſt number given 10; and the ſum 22 will be the firſt part of the ſecond number given 25, and conſequently the other part will be 3, which being multiplied by the ſecond number found 7, and the firſt part 22, being multiplied by the firſt number given 2, the ſum of their two products 21, 44, makes likewiſe 65.

Laſt of all, to find the firſt part of the third number given 30, multiply 5, the difference of the two laſt numbers given 25, 30, by the greateſt number found,

found, 7, and divide the product 35 by 5, the difference of the two numbers found 2, 7; then add the quotient 7 to 22, the firſt part of the ſecond number given 30, and the ſum 29 will be the firſt part of the third number given 30, and conſequently the other part will be 1, which being multiplied by the ſecond number found 7, and the firſt part 29 being multiplied by the firſt number found 2, the ſum of the two products 7, 58, makes ſtill 65.

Or elſe multiply 20, the difference of the firſt and the third number given, by the greateſt number found 7, and divide the product 140 by 5, the difference of the two numbers found 2, 7; then add the quotient 28 to 1, the firſt part of the firſt number given 10, and you will have 29 as above, for the firſt part of the third number given 30.

If you take 1 and 6 for the two numbers ſought for, and 4 for the firſt part of the firſt number given 10, in which caſe the other part will be 6, which being multiplied by the ſecond number found, 6, and the firſt part 4 by the firſt number found 1, the ſum of the two products 36 and 4, is 40. Upon this ſuppoſition, I ſay, the firſt part of the ſecond number given 25, will be 22, and conſequently the other part 3, which being multiplied by the ſecond number found 6, and the firſt part 22 by the firſt found number 1, the ſum of the two products 18, 22, is likewiſe 40; and in fine, the firſt part of the third number given 30, will be 28, and the other 2, which being multiplied by the ſecond number given 6, and the firſt 28 by the firſt 1, the ſum of the two products is ſtill 40. This Sport directs us to the ſolution of the following Queſtion.

Queſtion. *One woman ſold at market 10 apples at a certain rate apiece; another ſold 25 at the ſame rate; and a third ſold 30 ſtill at the ſame*

price;

price; and yet each of them brought the same sum of money home with them. The question is how this could be?

It is manifest, that to save the possibility of the Question, the women must sell their apples at two different sales, and at two different rates, seeing at each sale or division they sell at the same rate. Let the two different rates be 2 and 7, which are the two numbers that we found in the foregoing Sport; and we will suppose.

	Apples		*Farthings*	*Apples*		*Farth.*	
X.	1	at	2	9	at	7	
XXV.	22	at	2	3	at	7	65
XXX.	29	at	2	1	at	7	

that at the first sale they sold at 2 farthings an apple, and that at this rate the first sells 1 apple, the second 22, and the third 29; the three numbers 1, 22, 29, being the first parts of the three given numbers X, XXV, XXX, which were found in the foregoing Sport; in this case the first woman will take 2 farthings, the second 44, and the third 58. In the next place, if we suppose they sell the rest of their apples at 7 farthings, then the first woman will take 63 farthings for the 9 apples she had left, the second will take 21 farthings for the 3 apples she had left, and the third 7 farthings for the 1 apple she had left; and so each of them will take in all 65 farthings.

Or, if you will, make the two different rates 1 and 6, which were the two numbers found in the last Sport; and suppose at the first sale they sell at a farthing an apple, at which price the first sells 4, the second 22,

	Apples		*Farthings*	*Apples*		*Farth.*	
X.	4	at	1	6	at	6	
XXV.	22	at	1	3	at	6	40
XXX.	28	at	1	2	at	6	

and

and the third 28; theſe three numbers 4, 22, 28, being the firſt parts of the given numbers X, XXV, XXX, which were found in the laſt Sport; the firſt woman will take 4 farthings, the ſecond 22, and the third 28. Then ſuppoſe again that they ſell the reſt of their apples at 6 farthings a piece, the firſt woman will take 36 farthings for the 6 apples ſhe had left, the ſecond 18 for the 3 apples ſhe had left, and the third 12 farthings for the 2 apples ſhe had left. And thus every one of them will take in all 40 farthings.

SPORT XXIX.

Many numbers which proceed from 1 or unity, in a progreſſion, according to the natural order of numbers, (ſuch as theſe, 1, 2, 3, 4, 5, 6, &c.) being placed in a round form like a ring; to diſcover which of theſe numbers any one has thought upon.

LET any multitude of numbers in the aforeſaid progreſſion, ſuppoſe theſe ten, to wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, be marked upon ten ivory-counters (or for want thereof upon ten ſmall pieces of paper) which may be repreſented by theſe ten letters, A, B, C, D, E, F, G, H, K, L, *viz.* ſuppoſe 1 to be writ upon the counter A, 2 upon B, 3 upon C, *&c.* Then having placed thoſe counters circularly as you ſee (with their blank faces uppermoſt, and the figures underneath, that the ſubtilty

A
1
L 10 2 B
K 9 3 C
H 8 4 D
G 7 5 E
6
F

of

f the ſport may the better be concealed) let any ne think upon any number of units which does not xceed 10; that done, bid him touch one of thoſe ounters at pleaſure, and to the number on the ack-ſide of the counter touched (which you cannot e ignorant of, having noted well the place of 1 or) add ſecretly in your mind, the juſt number of ll the counters and reſerve the ſum; then bid him magine in his mind the counter touched to be the umber which he thought, and from that counter o count backwards, until he has made up the foreſaid ſum, which you reſerved, ſo will his computation infallibly end on the counter upon which he number thought of is marked.

For example, ſuppoſe he thought 7 or G, and hat he touched B, to wit, 2; add to 2 the number f all the counters, to wit, 10, ſo the ſum will be 2; then bid him count to 12, beginning at B and oing backward, and eſteeming B to be the number hought, to wit, 7; ſo will 8 fall upon A, 9 upon , 10 upou K, 11 upon H; and laſtly, 12 upon the ounter G, which being turned up will ſhew the umber thought.

The reaſon of this rule is not difficucult to be apprehended, two principles being preſuppoſed, the ne is this; to wit, many counters or things whatſoever being diſpoſed orderly one after the other, in ne continued line, whether it be right or circular; f you value or name the firſt counter to be ſome umber of units at pleaſure, and continue the count orward according to the natural order of numbers, ntil another number be named which falls upon he laſt counter: Or, if you imagine or name the aſt counter, to be the ſame number of units as efore you put upon the firſt, and continue to count ackwards to the firſt counter; I ſay that the ſame umber will be named at the end of both theſe com-

computations: For example, in theſe 9 letters, A, B. C. D. E. F. G. H. K. if the letter A be eſteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers, the letter K will fall upon the number 12. In like manner, if you eſteem K to be 4, and count backwards from K to A, the letter A will likewiſe fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12.
A.	B.	C.	D.	E.	F.	G.	H.	K.
12.	11.	10.	9.	8.	7.	6.	5.	4.

The other principle is this, to wit, many counters being diſpoſed in a round form like a ring; if you eſteem any one of thoſe counters to be ſome number at pleaſure, and then from that counter if you count circularly, until you end upon the counter where you began, the number laſt named will be equal to the ſum of the number of all the counters, and of the number which you put upon the firſt counter; for example, if D be one of ten letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural

A
1
L 10 2 B
K 9 3 C
H 8 4 D
G 7 5 E
6
F

progreſſion of numbers till you end with D where you began; the number 17, which is compoſed of 10 and 7, will neceſſarily fall upon D; for 9 (which is the number of letters in the circumference beſides

D)

D) being added to 7 (which was firſt put upon D) makes 16, to which 1 being added, (becauſe D ends as well as begins the circumference) the ſum is 17.

Now theſe two principles being preſuppoſed, it will not be difficult to apprehend the reaſon of the aforeſaid Rule in all caſes that can happen; for imagine that one has thought upon 7, or the counter G, then that counter which he ſhall touch muſt either be the ſame counter G, or ſome other that precedes or follows G.

Firſt therefore, ſuppoſing the counter or number touched to be the ſame with the number thought, the truth of the Rule will be then evident; for by the Rule given, he will begin to count from the ſame G to 17, putting 7 upon G, therefore by the ſecond preſuppoſition the number 17 will fall upon G.

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10; then according to the Rule adding 10 (the numbers of all the counters placed circularly) to 10 or L, the (counter touched) bid him count backwards to 20 by beginning at L, and eſteem L to be 7. Now, becauſe by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L; therefore by the firſt preſuppoſed principle, if we eſteem L to be 7, and count backwards, the number 10 will infallibly fall upon G, and then the number 20 ſhall alſo fall upon the ſame G by the ſecond preſuppoſed principle.

Laſtly, imagine he touched ſome number or counter which precedes 7 the number ſought, as B or 2; then adding 10 to 2, you are to bid him count unto 12, he having firſt imagined B to be the number thought 7; and going backward to A,

L, K, *&c.* Now becauſe by proceeding to count at B, which is 2, and beginning to count forward to C, D, *&c.* the number 7 falls upon G; therefore if one imagine that G is 2, and from thence count backwards towards F, E, *&c.* the number 7 will fall upon B (by the firſt preſuppoſed principle;) therefore when one aſſumes B to be 7, and counts towards A, L, *&c.* to any aſſigned number, it is in effect as much as when one imagines G to be 2, and counts towards F, E, unto the ſaid aſſigned number, for each of thoſe computations will end in the ſame point; but it is manifeſt (by the ſecond preſuppoſed principle) that eſteeming G to be 2, and counting towards F, E D, *&c.* round the whole circumference; the number 12 will fall upon the ſame G. And becauſe G being ſuppoſed to be 2, and counting on the ſame coaſt as before, the number 7 falls upon B; therefore if the computation be continued on the ſame coaſt as before, the number 7 falls upon B; therefore if the computation be continued on the ſame coaſt from B 7, to 12, the number 12 will fall upon the ſame G. So that the practice of this ſport in all its caſes is demonſtrated.

Note, That to the number of the counter touched you may not only add the number of all the counters once (as the Rule directs) but twice, thrice or more times: For example, B being touched, you may cauſe him to count to 12, or 22; or to 32, 42, *&c.* the reaſon whereof is evident from the ſecond preſuppoſed principle.

SPORT XXX.

Many numbers being ſhewed by pairs, to wit, two by two unto any perſon, that he may think upon any one of theſe pairs at pleaſure, to diſcover the pair that was thought upon.

LET 20 numbers, ſuppoſe theſe, 1, 2, 3, 4 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 17

17, 18, 19, 20, be writ upon ivory counters (or for want thereof upon ſmall pieces of paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, *&c.* Then diſpoſe them into pairs as you ſee, *viz.* ſuppoſe 1 and 2 to be one pair, 3 and 4 to be another pair, *&c.* and of theſe pairs let any

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20

one think upon which pair he pleaſes. That done, you are to diſtribute the ſaid 20 numbers into ranks, in form of a long ſquare, until there be 5 numbers in length, and 4 in breadth, after this manner, *viz.* lay the three firſt numbers, 1, 2, and 3, in a rank (as you ſee in the ſecond figure) from A towards B; then place 4 under 1, and 5 after 3, (in the ſaid rank A B) Again, place 6 under 4, and 7 after 5, (in the ſaid rank A B.) Then place 8 under 6,

alſo 9, 10, 11, on the right-hand of 4, in the rank C D. Again, place 12 under 9, and 13 on the right hand of 11, in the rank C D, and 14 under 12. Moreover, place 15, 16, 17, on the right hand of 12, in the rank E F. Laſtly, place 18, 19, 20, on the right-hand of 14, in the rank G H, ſo will all the numbers be ranked as you ſee in the Table. That done, you are to demand of him that thought upon two numbers as aforeſaid, in what rank or ranks the ſaid numbers happen to be found, *viz.* in which of the ranks A B, C D, E F, G H,

A	1	2	3	5	7	B
C	4	9	0	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

or in which two of the ſaid ranks: Now if he anſwer, that the two numbers he firſt thought upon are in the firſt rank A B, then 1 and 2 will be the numbers thought or kept in mind; if in the ſecond C D, then 9 and 10 ſhall be the numbers thought of; if in the third rank E F, then 15 and 16 will be the numbers thought: If they are in the fourth rank G H, then 19 and 20 ſhall be the numbers thought; but if he ſay that the numbers thought are in different ranks, then you are heedfully to mark the ſaid numbers 1 2, 9 and 10, 15 and 16, 19 and 20, which may be called the keys of the ſport, in regard they ſerve not only to diſcover the two numbers thought, when they are both in one and the ſame rank (as aforeſaid;) but even when they are in two different ranks: For in this latter caſe, as ſoon as it hath been declared to you in which two ranks the two numbers thought are placed, you

are

are to take the key of the higheſt of thoſe two ranks, and deſcending in a down-right line from the firſt number of that key to the lower of the ſaid two ranks, you'll there find one of the two numbers thought, and upon the right-hand of the ſecond number of the ſaid key, at the ſame diſtance ſideways, from the ſecond number of the key, (as one of the numbers thought was diſtant from the firſt number of the key,) you will find the other number thought.

For example, ſuppoſe the two numbers thought are 7 and 8, and it is declared to you, that they are in the firſt and fourth ranks; take then the key of the higheſt of thoſe two ranks; to wit, of the firſt, which is 1 and 2, and deſcending downright from 1 to the fourth rank, you'll there find 8 one of the numbers thought: then ſeek ſideways on the right-hand of 2 (the ſecond number of the key) a number as far ſeperated from 2, as 8 is diſtant from 1, and you'll find 7 the other number thought.

Again, ſuppoſe he ſays that the numbers thought are in the ſecond and third ranks: Take then the key of the ſecond rank which is 9 and 10, and deſcending down-right from 9 to the third rank, you ſhall there find 12, which is one of the numbers thought; then ſeek ſideways on the right-hand of 10, (the ſecond number of the key a number as far diſtant from 10, as 12 is from 9, and you'll find 11, which is the other number thought.

The reaſon of this will be apparent, from a ſerious conſideration of the placing of the numbers according to the Rules before given: for it is thereby evident, that of the firſt numbers coupled two by two, there can never be found more than one pair in one and the ſame rank; and of all the other pairs, one number is always found in one rank, and the other number in another rank.

Note alſo, that this ſport may be practiſed with divers perſons at once, and not only with 20 numbers, but with any ſuch multitude of numbers as is produced by the multiplication of any two numbers that differ by 1, or unity; as 30, which is the product of 5 multiplied by 6, and 42, which is the product of the multiplication of 6 and 7. That which is chiefly to be regarded is, the placing of the numbers in ranks according to the directions before given: And for the more eaſy comprehending of that order, I have, in the following Table, ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer illuſtration than can be expreſſed by many words.

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

SPORT XXXI.

Among three perſons, to find how many cards or counters each of them has got.

LET the third perſon take what number of cards or counters he pleaſes, provided it be evenly even, that is, diviſible by 4; let the ſecond take as many 7's as the other has taken 4's; and the firſt as many 13's. Then bid the firſt give to the other two as many of his counters as each of them had before; and the ſecond to give to the remaining two as many of his counters as each of them;

them; and in like manner, the third to give to each of the other the ſame number that they have. By this means it will ſo fall out, that they will all have the ſame number of counters, and each of them will have double the number that the third had at firſt. And for this reaſon, if you aſk one of the three how many counters he has got, half his number is the number the third had at firſt; and if you take as many 7's, and as many 14's as there were 4's in the third perſon's number, you will have the number of cards or counters that the ſecond and firſt took.

For example, if the third took 8 cards, it behoved the ſecond to take 14, that is, twice 7, becauſe there is twice 4 in 8; and the firſt muſt take 26, that is twice 13 by the ſame reaſon. If the firſt, who has 26 cards, gives to the ſecond 14, that is, as many as he had at firſt: and to the third 8, that being his firſt number, he will have only 4 left to himſelf; and the ſecond will have 28; and the third 16. But if the ſecond, who has 28 cards, gives out of his cards 4 to the firſt, who had juſt as many before, and 16 to the third, who had likewiſe as many; he will have 8 left to himſelf, and the firſt will have 8, and the third 32. In fine, if the third, who has got 32, gives 8 to each of the others, all the three will have 16, which is the double of 8, the number that the third took up at firſt.

1ſt	2d.	3d.
26	14	8
4	28	16
8	8	32
16	16	16

SPORT XXXII.

Of three unknown cards, to find what card each of three perſons has taken up.

THE number of each card taken up muſt not exceed 9. Then, to find out that number, bid

bid the firſt ſubtract 1 from double the number of the points of his card, and after mtltiplying the remainder by 5, add to the product the number of the points of the ſecond perſon's card. Then cauſe him to add to that ſum 5, in order to have a ſecond ſum; and after he has taken 1 from the double of that ſecond ſum, make him to multiply the remainder by 5, and add to the product the number of the points of the third perſon's card. Then aſk him the ſum ariſing from this laſt addition; for if you add 5 to it, you will have another ſum compoſed of three figures, the firſt of which towards the left is the number of the points of the card that the firſt perſon took up; the middle figure will be that of the ſecond perſon's card; and the laſt towards the right directs you to the third perſon's card.

For example; if the firſt took a 3, the ſecond a 4, and the third a 7; by taking 1 from 6, the double of the firſt 3, and multiplying the remainder 5 by 5, we have 25 product, to which we add 4, the number of the ſecond perſon's card, which makes 29, and that, with the addition of 5, makes the ſecond ſum 34, the double of which is 68, and taking 1 from that, there remains 67, which being multiplied by 5, makes 335, and this by the addition of 7, the number of the third perſon's card, and 5 over and above, makes the laſt ſum 347, the three figures of which ſeverally repreſent the number of each card.

Another way, Or, if you will, you may bid the firſt add 1 to the double of the number of the points of his card, and multiply the ſum by 5, and add to the product the number of the ſecond perſon's card. Then bid him add in like manner 1 to the double of the preceding ſum, and multiply the whole by 5, and add to the product the number of the third perſon's card. Then aſk him the ſum

ariſing

riſing from the laſt addition, and ſubtract 55 from r, that ſo there may remain a number compoſed of hree figures, each of which repreſents, as above, he number of each card.

As in the foregoing example, by adding 1 to 6 he double of 3, the number of the firſt perſon's ard, and by multiplying the ſum 7 by 5, we have ;5, which, with the addition of 4, the number of he ſecond peaſon's card, makes 39, the double of vhich is 78, to which if we add 1, and multiply the ım 79 by 5, we have 395; to that we add 7, the umber of the third perſon's card, and ſo have o2, from which if we ſubtract 55, the remainder s 347, the three figures of which ſeverally repreſent he number of each card.

SPORT XXXIII.

f three cards known, to find which and which is taken up by each of three perſons.

OF the three known cards, we ſhall call one A, the other B, and the third C, and leave each f the three perſons to pitch upon 1 of the three, hich may be done ſix different ways, as you ſee in the annexed ſcheme. Give the firſt perſon the number 12, the ſecond 24, the third 36. Then direct the firſt perſon to add together the half of the number of that perſon that has taken the card A, the third part of the number of the perſon that takes the card B, and the fourth part of the number of he perſon that takes the card C; and then aſk im the ſum, which you will find to be either 23, r 24, or 25, or 27, or 28, or 29, as you ſee in the

ſt.	2d.	3d.	Sums.
2	24	36	
A	B	C	23
A	C	B	24
B	A	C	25
C	A	B	27
B	C	A	28
C	B	A	29

the table or ſcheme, which ſhews, that if the ſum is, for example, 25, the firſt will have taken the card B, the ſecond the card A, and the third the card C; and if the ſum is 28, the firſt has taken the card B, the ſecond the card C, and the third the card A; and ſo on in the other caſes.

SPORT XXXIV.

To find out among ſeveral cards, one that another has thought of.

HAVING taken out of a pack of cards a certain number of cards at pleaſure, and ſhewn them in order upon the table, before the perſon that is to think, beginning with the lowermoſt, and laying them cleverly one above another, with their figures and points upwards, and counting them readily, that you may find out the number which, for example, we ſhall here ſuppoſe to be 12; bid him keep in mind the number that expreſſes the order of the card he has thought of, namely, 1, if he has thought of the firſt, 2, if he has thought of the ſecond, 3, if he has thought of the third, &c. Then lay your cards one after another, upon the reſt of the pack, in a contrary ſituation, and putting that upon the pack firſt that was firſt ſhewn upon the table, and that laſt that was laſt ſhewn. Then aſk the number of the card thought of, which we ſhall here ſuppoſe to be 4; that is, the fourth card in order of laying down, is the card thought of. Lay your cards, with their faces up, upon the table, one after another, beginning with the uppermoſt, which you are to reckon 4, the number of the card thought of; ſo the ſecond next to it will be 5, and the third under that 6, and ſo on till you come to 12, the number of the cards you firſt pitched upon to ſhew the perſon; and

nd you will find the card that the number 12 falls o, to be the card thought of.

SPORT XXXV.

everal parcels of cards being propofed or fhewn, to as many different perfons, to the end that each perfon may think upon one, and keep it in his mind; to guefs the refpective card that each perfon has thought of.

WE will fuppofe there are 3 perfons, and 3 cards fhewn to the firft perfon, that he may hink upon one of them, and thefe three cards laid fide by themfelves; then 3 other cards held before he fecond perfon, for the fame end, and laid apart; nd at laft, 3 different cards again to the third erfon, for the fame end, and likewife laid apart. This done, turn up the 3 firft cards, laying them n three ftations; upon thefe three lay the next hree other cards that were fhewn to the fecond erfon; and above thefe again the three laft cards. Thus you have your cards in three parcels, each of which confifts of 3 cards. Then afk each perfon in what lift is the card he thought of; after which it will be eafy to diftinguifh it; for the firft perfon's ard will be the firft of his heap; and in like maner the fecond's will be the fecond in his; and the hird perfon's card will be the third in his.

SPORT XXXVI.

everal cards being forted into three equal heaps, to guefs the card that one thinks of.

IT is evident that the number of cards muft be divifible by 3, fince the three lifts are equal. uppofe then there are 36 cards, by confequence here are 12 in each lift: afk in what lift is the ard thought upon; then put all the heaps together,

fo

ſo as to put that which contained the card thou
upon between the other two; then deal off the
cards again into three equal h nds, obſerving t
order of the firſt card to be firſt, the ſecond to
ſecond, the third to the third, the fourth to the f
again, and ſo round, dealing one card at a tin
till the cards are dealt off. Then aſk again,
what hand or heap is the card thought upon, a
after laying together the cards, ſo as to put that
which contained the card between the other tv
deal off again, as you did before, into three eq
liſts. Thus done, aſk once once more, what
the card is in, and you will eaſily diſtinguiſh whi
is it, for it lies in the middle of the liſt to which
belongs; that is, in this example, it is the ſix
card; or, if you will, to cover the artifice t
better, you may lay them all together, as befo
and the card will be in the middle of the who
that is, the eighteenth.

SPORT XXXVII.

To gueſs the number of a card drawn out of a co plete ſtock.

AFTER one hath drawn what card he pleaſ
out of a complete ſtock of 52 cards, for i
ſtance, ſuch as we play at Ombre with, you m
know how many points are in the card thus draw
by reckoning every fac'd card 10, and the reſt a
cording to the number of their points; then lookin
upon the reſt of the cards one after anoher, add th
points of the firſt card to the points of the ſecon
and the ſum to the points of the third, and ſo o
till you come to the laſt card, taking care all alon
to caſt out 10, when the number exceeds it; upo
which account you ſee it is needleſs to reckon in th
10's or the faced cards, ſince they are to be ca
out however. Then if you ſubtract your laſt ſu
fro

om 10, the remainder is the number of the drops
the card drawn.

It is easy to know that when nothing remains,
e card drawn is either a 10 or a faced card: and
at in this case, if it be a faced card, one cannot
stinguish whether it be king, queen or knave:
ow, in order to be master of that distinction, the
st way is to make use of a stock of 36 cards on-
, such as we formerly used for Piquet, and reckon
knave 2, a queen 3, and a king 4.

If you make use of a stock of 32 cards only, such
is now used for Piquet, you are to follow the
me course as is above prescribed, only you must
ways add 4 to the last sum, in order to have an-
ther sum, which being subtracted from 10 if it
e less, or from 20, if it surpasses 10, the remain-
er will be the number of the card drawn; so that
2 remains it is a knave, if 3 a queen, if 4 a
ing, &c.

If the stock is not full, you must take notice
hat cards are wanting, and add to the last sum
he number of all the cards that are wanting, after
btracting from that number as many 10's as are
be had; upon which, the sum arising from this
ddition is to be subtracted, as above, from 10 or
om 20, according as it is above or under 10.
his done, is is evident by casting your eye once
ore upon the cards, you may tell what card was
rawn.

SPORT XXXVIII.

To guess the number of the points or drops of two cards drawn out of a complete stock of cards.

LET a man draw at pleasure two cards out of a stock of 52 cards; bid him add to each of
he cards drawn as many other cards as his num-
er is under 25, which is the half of all the cards,
wanting

wanting 1, fixing upon each faced card what number he pleases; as if the first card be 10, add to 15 cards; and if the second card be 7, add to 18 cards; so that in this example there will remain but 17 cards in the stock, the whole number taken out amounting to 35. Then taking the remainder of the pack into your hands, and finding they are but 17, conclude that 17 is the joint number of all the points of the two cards drawn.

To cover the artifice the better, you need not touch the cards, but order the drawer to subtract the number of the points of each of the two drawn cards from 26, which is half the number of all the cards, and direct him to add together the two remainders, and acquaint you with the sum, to the end you may subtract it from the number of the whole stook, *i. e.* 52; for the remainder of that substraction is what you look for.

For *Example.* Suppose a 10 and a 7 are the cards drawn; take 10 from 26, there remains 16 and taking 7 from 26, the remainder is 19: the addition of the two remainders 16, 19, makes a sum of 35, which subtracted from 52, leaves 17 for the number of the drops of the two cards drawn.

The same is the management in a stock of 36 or 32 cards; only to colour the trick the better, instead of 26, the half of the cards, when they make 52, take another lesser number, but greater than 10, as 24, from which taking 10 and 7, there remains 14 and 17, the sum of which, 31, being subtracted from 52, the sum of all the cards, leaves 21 the remainder; from which subtract again 4, which is the double of the excess of 26 above 24, and so the remainder is 17, the number of the points of the two cards drawn, *viz.* 10 and 7.

If

If you make uſe of a Piquet-ſtock, conſiſting of 6 cards, inſtead of 18, the half of 36, the num-er of all the cards, take in like manner a leſſer umber, ſuch as 16, from which take 10 and 7, nd there remains 6 and 9, the ſum of which, 15, eing ſubtracted from 36, the number of all the ards, leaves 21 remaining; from which ſubtract gain 4, the double of the exceſs of 18 above 16, nd ſo the 17 remaining is the number of the points f the two cards drawn.

In like manner, if this Piquet-ſtock conſiſts only f 32 cards, inſtead of 16, the half of 32, the umber of the whole ſtock, take any leſſer number ou will, provided it be greater than 10, ſuch as 14, from which take 10 and 7, and the remainders re 4 and 7, the ſum of which, 11, being taken rom 32, leaves 21, and taking from that 4, the ouble of the exceſs of 16 above 14, you have 17 emaining, the number of the points of the 10 and the 7 drawn.

SPORT XXXIX.

To gueſs the number of all the drops of three cards drawn at pleaſure out of a complete ſtock of cards.

TO ſolve this Sport as the former, after the ſhorteſt way, the number of cards contained in the ſtock muſt be diviſible by 3; ſo that neither a ſtock of 52, nor one of 32, are proper; but one of 36 is, in regard 36, the number of all the cards, has 12 for its third part, which will aſſiſt us in the ſolution of the queſtion, as follows—

Let a man draw at pleaſure three cards out of a Piquet-ſtock of 36 cards; bid him add to each of theſe cards as many other cards as the number of their points falls ſhort of 11, which is the third part of the number of all the cards, wanting, on , allotting, as in the foregoing Sport, to each

faced

faced card what number he pleases: as if the first card is 9, he adds to it 2 cards; if the second is 7, he adds to it 4; and if the third is 6, he adds 5; which make in all 14 cards; so that in this example the remainder of the whole stock is 22 cards, which denotes the number of all the points of the three cards drawn.

The better to colour the artifice, you need not touch a card, but bid him subtract the number of the points of each of the three drawn cards from 12, the third part of 36, the number of the whole stock, and add together the three remainders, and tell you the additional sum, which you are to subtract from 36, and the remainder of that subtraction is what you look for.

As in this Example—Suppose he drew a nine, a seven and a six; take 9 from 12, there remains 3; take 7 from 12, there remains 5; and take 6 from 12, there remains 6; add the three remainders, 3, 5, 6, the sum is 14, which taken from 36 leaves 22 for the number of the drops of the three cards drawn.

To colour the trick the better, and to apply the rule to a stock that consists of fewer or more than 36 cards, such as one of 52 cards, make use of a number greater than 10, and lesser than 17, the third part of 52; for instance 15: bid him who drew the three cards add to each of his drawn cards as many other cards as the number of their respective points is under 15: for example, if the first card be 9, he adds to it 6 cards, if the second is 7, he adds 8; if the third is 6, he adds 9; which makes in all 26 cards; so that in this example there will remain in the main stock 26 cards. Taking the main stock into your hands, and finding you have 26 cards, subtract from 26 the number 4, which is the excess of 52, the number of the whole stock, above

above the triple of $15+3$, *i. e.* 48; and the remainder 22 is the number of all the points of the three cards drawn.

Or elſe you need not touch the cards, but bid the perſon that draws ſubtract the number of the drops of each of the three cards drawn, from 16, which is 1 more than the firſt number 15, and add together all the remainders, and acquaint you with the ſum; then do you ſubtract that ſum from the number abovementioned, 48, and you will find the remainder to be the number of all the points of the three cards drawn.

For Example—Suppoſe he drew a 9, a 7, and a 6; take 9 from 16, there remains 7; take 7 from 16, there remains 9; take 6 from 16, there remains 10; add theſe three remainders, 7, 9, 10, the ſum is 26, which ſubtracted from 48, leaves 22 for the number of the points of the three cards drawn.

In like manner, in a pack of 36 cards, take a larger number than 10, for inſtance 15; and taking notice of the additional cards, which amount to 26, as you ſaw but now, ſubtract that number, 26, from 36, the number of the whole pack, and to the remainder 10 add 12, which is the exceſs of the triple of $15+3$, *i. e.* 48, above 36, the number of the whole; and you will find the ſum 22 to be the number of points enquired after. In a Piquet-pack of 32 cards, inſtead of 12 you muſt add 16, by reaſon that 16 is the remainder of 32 ſubtracted from 48.

In imitation of this and the foregoing Sport it will be eaſy to ſolve the queſtion upon four or more cards drawn.

SPORT XL.

The Game of the Ring.

THIS is an agreeable game in a company of ſeveral perſons, not exceeding 9 (unleſs you have a mind to it) in order to the eaſier application

of the 22d Sport, *viz.* by reckoning the firſt perſon 1, the ſecond 2, the third 3, and ſo on; and in like manner, reckoning the right-hand 1, the left-hand 2; the thumb of the hand 1, the fore-finger 2, the third finger 3, the fourth 4, and the little one 5; the firſt joint 1, the ſecond 2, and the third 3. For if you put the ring to one in the company, for inſtance, the fifth perſon, and that upon the firſt joint of the fourth finger of the left hand, it is evident, that in order to gueſs who has the ring, and upon which hand, which finger, and whice joint, one has only theſe four numbers to gueſs, 5, 1, 4, 2, the firſt number 5 repreſenting the fifth perſon; the ſecond 1, the firſt joint; the third 4, the fourth finger; and the laſt 2, the left-hand. Now this is performed by obſerving the laſt method of Sport 22 foregoing, as appears from the following operation.

Taking 1 from 10, the double of the firſt number 5, and multiplying 9, the remainder by 5, you have 45; adding to that the ſecond number 1, you have 46, to which if you add 5, you have 51 for a ſecond ſum: the double of this ſecond ſum is 102, from which take 1, there remains 101, which being multiplied by 5, makes 505, and that with the addition of 4, the third number, makes 509 to which if you add 5, you have this ſecond ſum 514, the double of this 1028 leſſened by 1, and the remainder multiplied by 5 makes 5135, to which adding the fourth humber 2, you have thi ſum 5137, and that augmented by 5 gives this ſecond ſum 5142, the four figures of which repreſent the four numbers inquired for, and by conſequenc denote that the ring is upon the firſt joint of th fourth finger of the left hand of the fifth perſon.

PAR

PART II.

Artificial *Fire-Works.*

SPORT XLI.

To make Silent Powder, or ſuch as may be diſcharged without a noiſe.

THIS *unſounding powder*, if any ſuch there is, goes commonly under the name of White Powder, becauſe, poſſibly, the firſt made was of that colour. It is not probable it can be of any great force, for as much as the noiſe of gun-powder proceeds from the violent percuſſion of the air, occaſioned by the ſtrength of it. I have not indeed ſeen this powder myſelf, yet I have read in authors ſeveral ways of making the ſame, of which the following two only occur to my memory.

The firſt is thus. To one pound of common Gun-powder take half as much Venetian Borax, which having pulveriſed, mixed, and well incorporated together, reduce the mixture into grains, and you have the powder required.

The other way is—To four pounds of common Gun-powder add two pounds of Venetian Borax, one pound of Lapis Calaminaris, and one pound of Sal-armoniac; pulveriſe them all together, to make of them a powder in grains, as before.

SPORT XLII.

To prepare an Oil of Sulphur required in Fireworks.

HAVING melted what quantity of Sulphur you think fit, upon a moderate fire, in an earthen or copper veſſel, throw into it ſome old, or, in defect of this, ſome new brick, that is well burnt, and was never wetted, broken into many ſmall pieces about the bigneſs of a bean; ſtir them continually with a ſtick, till they have drunk up and conſumed all the ſulphur; this done, ſet them upon a furnace to diſtil in an Alembic; ſo you ſhall have a very inflammable oil, fit for your purpoſe.

You may make it otherways thus—Fill one third or fourth part of a glaſs-bottle with a long neck with Sulphur pulveriſed; then pouring upon it ſpirit of Turpentine, or oil of Walnuts, or of Juniper, till the bottle is half full, ſet it upon hot cinders, leaving it there eight or nine hours, and you ſhall find an oil therein of the aboveſaid quality.

SPORT XLIII.

To prepare the Oil of Saltpetre uſeful in Fireworks.

PUT upon a fir-board, well plained and dry, what quantity of purified Saltpetre you pleaſe, and cauſe it to melt by putting thereupon burning coals; and you ſhall ſee the liquor to paſs through the board, and to fall down drop by drop, which muſt be received in an earthen or copper pot, where you have an oil of Salt-petre, fit to be uſed in fireworks, as we ſhall declare in its proper place.

SPORT XLIV.

To prepare the Oil of Sulphur and Saltpetre mixed together.

HAVING mixed and well incorporated equal portions of Sulphur and Saltpetre, reduce all into a fine powder, which muſt be paſſed through a fine ſearce; put this powder thus ſearced into a new earthen pot, or one that hath not been uſed, and pour upon it good white-wine vinegar, or elſe brandy, till it is covered. Then cover your pot ſo that no air may get into it, and ſet it to ſtand in ſome hot place, till all the vinegar is conſumed or diſappears. Laſt of all, draw from the remaining matter the oil by means of an Alembic; which will ſerve to ſeveral purpoſes of Fireworks.

SPORT XLV.

To make Moulds, Rowlers, and Rammers for Rockets of all ſorts.

A Rocket, which the French call *Fuſee,* the Latins *Rochetta*, and the Greeks *Byrobolos*; conſiſts of a Cartouch or paper tube called the Coffin, and a combuſtible compoſition, with which it is loaded; which being fired, mounts into the air, in a manner moſt agreeable to behold.

There are three ſorts of them; the Small, the Middling and the Great. All ſuch are reckoned Small, whereof the diameters do not exceed that of a lead bullet of one pound, or whoſe moulds admit not a bullet above that weight. The Middleing are thoſe the moulds of which will admit bullets from one to three pounds weight. The Great will carry from a three pound to an hundred pound ball.

To determine the bigneſs of theſe Coffins to a required meaſure, that is, length and thickneſs, and to make any demanded number of them, of the ſame reach, and of equal force, they muſt be fitted to a concave cylinder, made of ſome hard matter, and turned exactly in a lathe: this is called the Mould or Form, which is ſometimes made of metal, but moſt commonly of hard wood, ſuch as Box, Juniper, Aſh, Cypreſs, wild Plum-tree, Italian Walnut-tree, and ſuch like.

Beſides this, there is another but a convex and ſolid cylender of wood required, called a Rowler, upon which the thick paper, whereof the Coffin is made, muſt be rowled, till it is of a bigneſs exactly to fill the concavity of the Mould. The diameter of this Rowler muſt contain five eighth parts of that of the Mould, the length of which muſt be ſix times the diameter of its bore, in ſmall rockets; but in the middling and the large ones it muſt be only five, or four times the length of the diameter of their bore.

Another cylinder of wood muſt alſo be had, which is to be a little ſmaller than the former, that it may go into the Coffin with the greater eaſe. And this is to ſerve for a Rammer, to drive down the Compoſition into the Coffin when you charge it. But firſt your Coffin muſt be ſtraitened or choaked, which is done by winding a cord about the end of it, after you have a little withdrawn the Rowler, turning in the mean time the Coffin, and drawing the cord, till there remains only a little hole, which then muſt be tied with ſtrong packthread. This done, you muſt draw out the Rowler, and introducing the Rammer into the Coffin, put all into the Mould; and when you have ſtruck five or ſix blows with a mallet upon the Rammer, to give a good form

form to the neck of the Rocket, the Coffin is finiſhed, and ready to be filled upon occaſion.

This Rammer muſt be bored lengthwiſe to ſome depth, that it may receive into its concavity the Needle which muſt be in the mould, together with the Coffin and Rammer, the uſe of this needle, which muſt be one third part of the length of the Coffin or Mould is to make a vent for the priming in the bottom of the Compoſition, of which we ſpeak in the enſuing Sport.

SPORT XLVI.

To prepare a Compoſition for Rockets of any ſize.

THE Compoſition wherewith the coffins are to be filled is different according to the different bigneſs of them; for it is found by experience, that what is fit for ſmall rockets, burns too violently, and too quickly in thoſe that are large, becauſe the fire is bigger, and the matter alſo driven cloſer together: hence it is that no gunpowder is uſed in the larger ſort. In making up this compoſition, according to the different ſizes of rockets, the following proportions muſt be obſerved.

For Rockets from 60 to 100 pounds, you muſt to three pounds of Saltpeter, add one pound of Sulphur, and two pounds of good Wood-coal.

If they are from 30 to 50 pounds, to 30 pounds of Saltpeter, put ſeven pounds of Sulphur, and ſixteen pounds of Coal.

Rockets from 18 to 20 pounds, to twenty one pounds of Saltpeter, require ſix of Sulphur and thirteen of Coal.

From 12 to 15 pounds, require to four pounds of Saltpeter one pound of Sulphur, and two pounds of Coal.

If they be from 9 to 12 pounds; to ſixty two

pounds of Saltpeter, add 9 pounds of Sulphur, and twenty of Coal.

From 6 to 9 pounds; add to ſeven pounds of Saltpetre, one of Sulphur, and two of Coal.

From 4 to 5 pounds; to eight pounds of Saltpetre, add one pound of Sulphur, and two of Coal.

From 2 to 3 pounds; to ſixty pounds of Saltpetre, add two of Sulphur and 15 of Coal.

For one pound; to ſixteen pounds of Gunpowder, add one pound of Sulphur, and three of Coal: or to nine pounds of Powder, four of Saltpetre, one of Sulphur, and two of Coal.

For twelve ounces; put to nine ponnds of powder, four of Saltpeter, one of Sulphur, and two of Coal.

For eight ounces; add to thirty pounds of Powder, twenty four of Saltpetre, three of Sulphur, and eight of Coal.

For five and ſix ounces; to thirty pounds of powder add twenty four pounds of Saltpetre, three pounds of Sulphur, and eight pounds of Coal.

For four ounces; add to twenty four pounds of powder, four pounds of Saltpetre, two pounds of Sulphur, and three pounds of Coal.

For two and three ounces; to twenty four pounds of powder, put four pounds of Saltpetre, one pound of Sulphur, and three pounds of Coal.

For an half ounce, and an ounce; take fifteen pounds of Powder, and two pounds of Coal.

For the ſmaller Rockets: to nine or ten pounds of Powder, add one pound, or one and a half of Coal.

Here follow alſo other proportions, which experience hath taught to ſucceed extremely well.

For Rockets that contain one or two ounces of matter. Add to one pound of Gunpowder, two ounces

ounces of good Coal: or, to one pound of Musket-powder, take one pound of coarse Cannon-powder: or, to nine ounces of Musket-powder, put two ounces of Coal: or to one once of Powder, an ounce and a half of Saltpetre, with as much Coal.

For Rockets of two or three ounces; add to four ounces of Powder, one ounce of Coal: or to nine ounces of Powder, two ounces of Saltpetre.

For a Rocket of four ounces; add to four pounds of Powder, one pound of Saltpetre, and four ounces of Coal, and if you please half an ounce of Sulphur: or to one pound two ounces and an half of Powder, four ounces of Sulphur, and two ounces of Coal: or to one pound of Powder, four ounces of Salpeter, and one ounce of Coal; or to seven ounces of powder, four ounces of Saltpetre, and as much Coal: or, add to three ounces and an half of Powder, ten ounces of Saltpetre, and three ounces and an half of Coal. The composition will be yet more strong, if it be made up of ten ounces of Powder, three ounces and an half of Saltpetre, and three ounces of Coal.

For Rockets of five or six ounces, take two pounds five ounces of Powder, to half a pound of Saltpetre, two ounces of Sulphur, six ounces of Coal, and two ounces of Filings of Iron.

For Rockets of seven or eight ounces; add to seventeen ounces of Powder, four ounces of Saltpetre, and three ounces of Sulphur.

For Rockets from eight to ten ounces; to two pounds five ounces of Powder, put half a pound of Saltpetre, two ounces of Sulphur, seven ounces of Coal, and three ounces of Filings.

For Rockets from ten to twelve ounces; take to seventeen ounces of Powder, four ounces of Saltpetre, three ounces and an half of Sulphur, and one ounce of Coal.

For

For Rockets from fourteen to fifteen ounces, t two pounds four ounces of Powder muſt be addec nine ounces of Saltpetre, three ounces of Sulphu five ounces of Coal, and three ounces of File duſt

For Rockets of one pound, to one pound Powder, take one ounce of Sulphur, and thre ounces of Coal.

For a Rocket of two pounds, add to one poun four ounces of Powder, twelve ounces of Saltpetr one ounce of Sulphur, three ounces of Coal, an two ounces of File-duſt of Iron.

For a Rocket of three pounds, to thirty ounce of Saltpetre, put ſeven ounces and an half of Su phur, and 11 ounces of Coal.

For Rockets of four, five, ſix, or ſeven pound add to thirty one pounds of Saltpetre, four poun and an half of Sulphur, and ten pounds of Coal.

For Rockets of eight, nine, or ten pounds, ta to eight pounds of Saltpetre, one pound four ou ces of Sulphur, and two pounds twelve ounces Coal.

The proportion of the different materials bei thus determined, each of them muſt be well beate and ſearced apart, and afterward weighed a mixed. Thus is your Compoſition ready wher withal to charge your Coffins, which muſt be ma of ſtrong paper well paſted.

SPORT XLVII.

To make a Rocket.

YOUR Coffins and different Compoſitions bei in readineſs, you muſt chuſe a Compoſiti ſuitable to the largeneſs of your deſigned Rocke which muſt neither be too wet nor too dry, but little moiſtened with ſome oily liquor, or with bra dy; then take your Coffin, the length of whi

mu

muſt be proportioned to the bigneſs of its concavity; put it, with the Rammer, into the Mould; then put into it ſome of your Compoſition, taking good care not to put in too much at a time, but only one ſpoonful or two; then put in your Rammer and with a Mallet ſuited to the bigneſs of the Coffin, ſtrike three or four ſmart blows directly upon it; then withdraw the Rammer again, and pour in an equal quantity of your Compoſition, and drive it down in like manner with your Rammer and Mallet, giving the ſame number of blows; continue thus doing till the coffin is filled to the height of the Mould, or rather a little below it, that five or ſix folds of the paper may be doubled down upon the Compoſition thus driven into the Coffin, which ſometimes inſtead of paper is made of wood.

The Coffin being filled with the mixture, and the paper doubled down upon it, you muſt beat it hard with the Rammer and Mallet to preſs down the folds of the paper, upon which you may put ſome Corn-powder, that it may give a report. In this paper folded down, you muſt make three or four holes with a bodkin, which muſt penetrate to the Compoſition, to ſet fire to the Stars, Serpents, and Ground-rockets, when ſuch there are; otherwiſe it will ſuffice to make one hole only, with a broach or bodkin, which muſt be neither too ſmall nor too great, but about one fourth of the diameter of the bore, as ſtraight as poſſible, and in the very middle, in order to fire the Corn-powder.

SPORT XLVIII.

To make Sky-Rockets that mount into the air with ſticks.

IT is to be noted, that the Head of a Rocket, is the higheſt end, by which it is loaded, and which riſes firſt when it is fired: the Neck of the Rocket,

Rocket, or its Tail, is the lower end, where it w
choaked or ſtraightened, and the priming is pu
which muſt be of good Corn-powder.

Your Rocket being charged, as was taught in th preceding Sport, you muſt have a long rod or ſtick of ſome light wood, ſuch a Oſier, or Fir, whic muſt be bigger and flat at one end, growing ſlen derer towards the other. This ſtick muſt be ſtraigh and ſmooth, without knots, and plained if nee be. Its length and weight muſt be proportione to the ſize of the Rocket, being ſix, ſeven, o eight times the length of it; to the larger end o this, where it is flatted, you muſt tie your Rocket its Head reaching a little beyond the end of th ſtick, and being thus fixed, lay it upon your finge two or three inches from the neck of the Rocket which ſhould then be exactly balanced by the ſtick if it is rightly fitted; after which you have nothing to do, but to hang it looſly, upon two nails, perpendicular to the horizon, with its head up, and then it is ready for firing. But if you would have it to riſe very high, and in a ſtraight line, you muſt put a pointed paper cap upon its head, and it will pierce the air with greater facility.

To theſe Rockets, for the greater diverſion of the ſpectators, ſeveral other things may be added: as Petards or Crackers, thus; get a box of iron ſoldered, of a convenient bigneſs, fill it with fine grain-powder; put it into the Coffin upon the compoſition, with the touch-hole down, double the reſt of the paper upon it to hold it faſt till the mixture is conſumed, and then firing it will give a report in the air.

You may add to them likewiſe, Stars, Golden-rain, Serpents, Fire-links, and other ſuch agreeable works, the making of which ſhall be taught afterwards. In order to this, you muſt have in readi-

neſs

nefs an empty Coffin, of a larger diameter than your Rocket. This muſt be choaked at one end, ſo as only to admit the head of the Rocket, to which it muſt be faſtened. Into this large Coffin, having firſt ſtrewed the bottom of it with Meal-powder, you muſt put your Serpents, or Golden-rain, or Fire-links, with the primed end downwards; and amongſt, and over your ſtars you muſt throw a little powder. Then you may cover this additional Coffin with a piece of paper, and fit to it a pointed cap as before, to facilitate its aſcention.

SPORT XLIX.

To make Sky-Rockets which riſe into the Air without a ſtick.

SKY-*Rockets* without Sticks muſt be ſmall, becauſe they are held in the hand, from whence they riſe, after you have put fire to the priming they are made as the foregoing; but that they may the better fly into the air, you muſt fit to them four wings diſpoſed Croſs-wiſe, like the feathers of darts or arrows; their length muſt be one third part of that of the rocket, their breath at the lower part half their length, and their thickneſs about a ſixth or eight part of the diameter of the orifice of the rocket.

Inſtead of four of theſe wings, you may uſe three of the ſame dimenſions with equal ſucceſs; but with this caution, that in placing them upon your rocket the lower ends of them muſt be let down below the tail of it the length of one diameter of its orifice. There are many other ways of making theſe rockets, according to the various fancies of artiſts, which would be too tedious for this work.

If the Compoſition for your Rockets is defective, as is known when they riſe, either not at all, or with

with difficulty, or fall down again before co
ſumption of the mixture; or when they mount n
with an equal and upright motion, but turning ar
winding, or whirling in the air; to amend yo
compoſition, you muſt diminiſh the quantity
coal when it is too weak, and add to it if too ſtron
as it is when it burſts the rocket, the coal ſervin
to abate the force of the powder, and to give
fine train to your rocket. Wherefore it would b
convenient, before you make up a quantity of rock
ets, to try your mixture and correct its faults.

To preſerve your rockets in good condition, the
muſt be kept in a place neither too dry, nor to
moiſt, but temperate; and the compoſition ſhoul
not be made up, but upon occaſion to uſe it. You
rocket muſt not be pierced, till you deſign to pla
it; which muſt not be in a ſeaſon of wind or rain
or when the nights are moiſt with fogs and miſts
all which are prejudicial to the agreeable effects o
a rocket.

If you would have your rocket to burn with a pal
white flame, mix ſome Camphire with your com-
poſition; inſtead of which if you take raſpings o
Ivory, the flame will be of a clear ſilver-colour, but
ſomewhat inclining to that of lead; if Colophony
or Grecian-pitch, it will be of a reddiſh copper-
colour; if black or common pitch, the flame will
be dark and gloomy; if Sulphur, it will be blue;
if Sal-armoniack, it will appear greeniſh; if crude
Antimony, or the raſpings of yellow Amber, it will
emit flames of a like colour.

SPORT L.

To make Ground Rockets, which run upon the earth.

Rockets that run along the Ground, called there-
fore Ground-rockets, require not ſo ſtrong a

compo-

compoſition, as thoſe that mount into the air; and therefore continue longer, burning as well as moving more ſlowly: Wherefore they vary from the others, as well in the demenſions of their Coffins, as in the compoſition wherewith theſe are charged. The length of the bore or concavity, may be eleven times that of its diameter; the rowler on which the coffin is made, may be five lines in diameter, and the rammer a little leſs, that it may go caſily into the coffin without ſpoiling it.

The Compoſition may be of cannon powder only, well beaten and ſearced till it is as fine as flower, wherewith you muſt fill the coffin, by little and little as before, within a finger's breath of the brim of the mould; then doubling down one third part of the paper, knock it down with the rammer and mallet, and after with a bodkin, make a ſmall hole which may penetrate to the compoſition; then put in a piſtol-charge of fine powder, doubling down ſome more of the paper upon it, the reſt of which muſt be choak'd tying it hard with pack-thread.

Theſe rockets being ſmall are charged only with powder finely pulverized, without any coal, herein differing from the large ones, that have no powder at all, except in their priming, which in both ſorts muſt be of well grained powder: The reaſon of which is, becauſe in a greater concavity there is a greater fire acting upon a greater quantity of matter, and conſequently with more violence; there being alſo a greater quantity of air to be rarified in a great than in a ſmall rocket.

When you choak or ſtraighten the end of your Rocket, whether ſmall or great, you muſt have a hook or ſtaple driven into a poſt, or into a wall, to this tie one end of your cord, which muſt be of a ſize proportionable to your rocket, or to the bar of

a win-

a window, and the other to a ſtrong ſtick, which you maſt put between your legs: thus the cord being winded about your rocket in the deſigned place, you may draw, turning, and ſtraightning it by degrees as you deſire.

SPORT LI.

To make Rockets that fly on a line, called Air-Rockets.

THIS is done with ordinary rockets that muſt not be too big, by faſtening to them two iron rings, or, which in my opinion is better, a wooden pipe or cane, thro' which muſt paſs a well ſtretched line: thus if you ſet fire to your rocket, it will run along the line without ceaſing till all the matter is ſpent.

If you would have yonr rocket to run back, as well as forward, after you have filled one half of the coffin with the compoſition, ſeperate this from the empty half by a wheel of wood fitted exactly to the cavity; in the middle of this wheel muſt be a hole, from which a ſmall pipe, filled with meal-powder, muſt paſs along the middle of the empty half, which then muſt be filled with the compoſition; and ſo after the firſt half of the rocket is conſumed, the fire being communicated by the little pipe, will light it at the other extremity, and ſo drive it back to the place from whence it came.

The ſame thing may be effected by the means of two rockets tied together, the tail of the one to the head of the other, one of which being burnt to the end, fires the other, making it to run back: but leaſt the ſecond ſhould catch fire at the head, it muſt be defended with a cover of paper or waxed cloth.

This ſort of rockets is commonly uſed to ſet fire to other machines in fire-works for diverſion, to which,

which, for the greater pleaſure, they give the figures of ſeveral animals, ſuch as ſerpents or dragons, which then are called Flying Dragons; and are extremely agreeable, chiefly when filled with ſeveral other works, as golden rain, hairs dipt in wild-fire, ſmall nut ſhells filled with the rocket compoſition, and many other diverting things, of which afterwards.

SPORT LII.

To make rockets that burn in the water, called Water-Rockets.

THO' the fire and water are oppoſite elements mutually deſtroying one another; yet the rockets we have hitherto diſcribed, being once lighted will continue to burn even in the water, and will have their full effect; but for as much as it is done under water, we are deprived of the pleaſure of beholding it. In order therefore, to make them to ſwim upon the water, we muſt alter ſomewhat the proportions of their mould, as well as the materials of their compoſition.

The mould, then, required to ſuch rockets, may be eight inches in length, and its bore an inch over. The rowler muſt be of nine lines diameter, and the rammer not quite ſo thick: no needle is required to this mould.

The compoſition, if you would have your rocket burn on the water with a clear flame like a candle, muſt be made of three ounces of powder beaten and ſearced, one pound of ſalt-petre, and eight ounces of ſulphur mixed together: when you deſire your rocket to appear on the water with a fine tail, you muſt, to eight ounces of common powder, add one pound of ſalt-petre, eight ounces of ſulphur, and two ounces of coal.

The compoſition being prepared, and the coffin charged with it, as is taught above, put a fire-link

at the end of it; and covering your rocket with wax, pitch, or rosin, to preserve the paper from the water, fasten to it a stick of white willow about two foot long, which will cause it to swim upon the water.

Many other different ways may such rockets be made without altering either the mould or composition, for which the curious may consult the authors that have writ particular treatises of Pyrotechny.

A rocket also may be made, which, after burning some time in the water, will throw up into the air sparkles and stars; which is done by dividing the rocket into two parts with a wooden wheel having a hole in the middle, one partition being filled with the common composition, the other with stars, having some powder strewed amongst them.

Moreover you may contrive a rocket, which, having burnt one half of its time in the water, will mount up in the air with great swiftness; thus: having filled two equal coffins with good composition, past them together slightly only at the middle, the head of the one answering the tail of the other; betwixt them must pass a little pipe at the extremity, to light the other, when one is consumed. Then fasten the rocket to which the other is joined, to a stick of such length and bigness as is required for balancing it, and to the lower end of the rocket tie a pack-thread, to which you must fasten a large musket-ball that must hang upon the stick by means of a bent wire. This done set fire to your rocket, it being in the water: and its composition being consumed, will light, by means of a little pipe, the other rocket, which will mount into the air, through the strength of the fire, the first being kept down by the weight it sustains.

SPORT LIII.

To make Fire-Links.

A *Fire-Link*, ſo called from its reſemblance to the links of a ſaucidge, is a kind of Rocket, that is uſually tied to the end of a bigger one, to render the effect more agreeable. I ſaid, uſually, becauſe there are ſome of them made that fly into the air as Sky-rockets, and are called Flying Fire-links, to diſtinguiſh them from the others which are named fixed Fire-links. We ſhall here briefly teach the making of both ſorts.

And firſt the fixed kind to be faſtened to a Rocket is made thus: Take a coffin of what bigneſs you think fit, and having choaked it at the end, fill it with fine Powder, and choak it at the other end; then roll it ſtrongly with ſmall cord from one end to the other, gluing the cord with good glue, to keep it faſt, and to ſtrengthen the Coffin, that it may give the greater noiſe when it breaks: thus is your fire-link ready to be faſtened to the end of a Rocket, either with paper, parchment, or cord, or otherwiſe; but note, that you muſt pierce the end of your Fire-link, which joins to the Rocket, and prime it with Corn-powder.

To make flying Fire-links, you muſt have ſuch Coffins as for the former, only they muſt be a little longer, and having choaked them at one end, charge them with corn-powder, adding at laſt meal-powder to the thickneſs of one inch, driving all down, as in ſky rockets, with a mallet. Then ſtrengthen the coffin with a line, as in the former, after you have choaked the other end, leaving a hole about the bigneſs of a gooſe-quill, to which you muſt put a little moiſtened powder for priming.

Or, having choaked at one end, and charged your coffin within one inch of the other end, choa

it there, leaving only a ſmale hole, which if quite ſhut up, or too ſmall, muſt be opened with a bodkin; then fill up your empty ſpace with powder finely flowered, or with the compoſition for ſky-rockets, which muſt be driven cloſe with a rammer and mallet, doubling down the remaining paper, if any, upon your compoſition, which will give a fine tail to your link; and when you have made a hole in the middle of this laſt paper, and primed it, your flying link is ready to be thrown into the air, which is done thus.

You muſt provide guns or cannons with a vent at bottom, where there muſt be a tail ſomewhat long, which muſt paſs through a piece of wood, that it may reach to a fire-conveyance running along underneath, to ſet fire to the cannons one after another, which will alſo throw up into the air the links with a noiſe in the ſame order.

SPORT LIV.

To make Fire-Lances.

LANCES of fire, are long and thick pipes or cannons of wood, with handles at the end, whereby they are made faſt to ſtakes or poſts, well fixed, that may ſuſtain the force of the fire, having ſeveral holes to contain rockets or petards. They are uſed in feſtival Fireworks that repreſent nocturnal fights, as well for throwing rockets, as making vollies of reports.

You muſt uſe them thus—Put a Rocket into every hole, and fill the bore of the cannon with Compoſition, which fired will, as it conſumes, fire the Rockets one after another, and throw them up into the air. But if you would have many thrown up at once, cover the bottom of the Lance with compoſition, and thereupon place a long ſmall pipe filled

filled with the ſame compoſition, about which put your Rockets, till you have filled your cannon, the primed end being downwards, that ſo firing the compoſition in the pipe, this may light that at the bottom of the Lance, which firing the Rockets, they will mount all at once into the air.

There may be many other ways of contriving Fire-lances in imitation of this, of which I ſhall not ſpeak: I ſhall only mention one other ſort of theſe Lances. This conſiſts of a Coffin made of ſtrong paper well glued, which may be of what dimenſions you think fit, according as it is deſigned to give more or leſs light; this muſt be filled with the Star-compoſition, pulveriſed and primed with Meal-powder moiſtened: the lower end muſt be ſtopped with a round piece of wood, which muſt appear two inches without the coffin, that thereby it may be faſtened at pleaſure.

Remark. The name of fiery or burning Lances, and Pikes, is alſo given to a kind of pikes like a javelin or dart, with a ſtrong iron-pointed head, called by the Latins *Phalorica*, and *Dardi di Fuoco* by the Italians, which were formerly thrown, being firſt fired againſt the enemies, either by the hand or from engines, being covered between the iron and wood with tow dipt in ſulphur, roſin, Jews' pitch, and boiling oil; where they lighted they ſtuck, ſetting on fire whatever was inflammable.

This ſort of Lances is not now in uſe, but inſtead of them we have Buruing Arrows, that are not leſs terrible, though not much now in eſteem; however, we will here gratify the curious with a brief deſcription of them. Flaming Arrows are artificial firebrands thrown amongſt the enemies' works, to reduce them to aſhes: they are made thus—Prepare a little bag of ſtrong courſe cloth,

 about

about the bigneſs of a gooſe's or ſwan's egg, of a globular or ſphæriodal figure, which muſt be filled with a compoſition made of four pounds of beaten Powder, as much refined Saltpetre, two pounds of Sulphur, and one pound of Grecian Pitch: or you may make it of two pounds of Meal powder, eight pounds of Saltpetre refined, two pounds of Sulphur, one pound of Camphire, and one pound of Colophony: or yet more ſimply thus—of three pounds of powder, four pounds of Saltpetre, and two pounds of Sulphur. With one of theſe mixtures fill the bag, preſſing it hard, and make an hole through the middle of it lengthwiſe, to receive an arrow, like thoſe of the ordinary bows or croſs-bows, the head of it remaining without the bag, which muſt be faſtened ſo as it may not move or ſlide towards the feathers. This done, roll your bag with ſtrong packthread as thick as poſſible from one end to another, and then cover it all over with Meal-powder mixed with melted pitch. Thus it is ready to be ſhot out of a bow or croſs-bow, after it is fired by two little holes made for that purpoſe near the head of your arrow.

SPORT LV.

To make Stars for Sky-rockets.

STARS are little balls about the bigneſs of a muſquet-bullet, or an hazle-nut, made of an inflamable compoſition, which gives a ſplendid light, reſembling that of ſtars, from whence is the name. When they are put into the rocket, they muſt be covered with prepared tow, the manner of making which ſhall be taught, after that of ſtars.

They are made thus: to one pound of powder finely flowered, add four pounds of ſalt-petre, and two pounds of ſulphur; and having mixed all very well, roll up about the bigneſs of a nutmeg of this

mixture

mixture in a piece of old linnen or in paper; then tie it well with pack-thread, and make a hole through the middle with a pretty big bodkin, to receive ſome prepared tow, which will ſerve for priming: this being lighted, fires the compoſition, which emiting a flame through both holes, gives the reſemblance of a pretty large ſtar.

If inſtead of a dry compoſition, you uſe a moiſt one in form of paſte, you need only roll it into a little ball, without wrapping it up in any thing, ſave, if you will, in prepared tow, becauſe of itſelf it will preſerve its ſpherical figure; nor needs there any priming, becauſe while moiſt you may rowl it in meal-powder, which will ſtick to it, and when fired will light the compoſition, and this at falling forms itſelf into drops.

Remark. There are many other ways of making ſtars, too long now to be mentioned; I ſhall only here ſhew how to make Stars of Report, that is, ſtars that give a crack like that of a piſtol or muſqnet, as follows.

Take ſmall links, made as is taught in Sport 53. which you may chooſe either to roll with line or not; tie to one end of them, which muſt be pierced, your ſtars if made after the firſt manner, that is, of the dry compoſition: otherwiſe you need only leave a little piece of the coffin empty beyond the choak of the pierced end, to be filled with moiſt compoſition, having firſt primed your vent with grain powder.

You may alſo contrive ſtars, which, upon conſumption of the compoſition, may appear to be turned into ſerpents, a thing eaſy to be performed by ſuch as underſtand what precedes; upon which account, and becauſe they are but little in uſe, I ſhall ſay no more of them.

SPORT LVI.

To make prepared Tow for priming to Fire-works.

PRepared Tow, called alſo Pyrotechnical Match, and Quick-match, to diſtinguiſh it from Common Match, is uſed for priming all ſorts of machines for Fire-woıks of diverſion, ſuch as rockets, fire-lances, ſtars, and the like; and it is made as follows.

Take thread of flax, hemp or cotton, and double it eight or nine times, if it is for priming your large rockets, or fiery lances; but four or five times only, if it is to be put through your ſtars. Having made it of a bigneſs proportioned to your deſigned uſe, and twiſted it, but not too hard, wet it in clean water, which muſt be after ſqueezed out with your hands. Then put ſome gun-powder in a little water, ſo a to thicken it a little; in this ſoak your match well, turning and ſtiring it till it is thoroughly impregnated with the powder; and then taking it out rowl it in ſome good powder duſt, and hang it upon lines to dry either in the ſun or ſhade: thus you have a Pyrotechnical Match ready for uſe on all occaſions.

Common Match, called alſo Fire-cord, is thus made: Take an unglazed earthen pot; cover its botom with red ſand well waſhed and dryed; upon this lay ſpiral-wiſe plain match of cotton, or well-cleaned tow, half an inch thick, the diſtance of half an inch being between each revolution, and then cover it with ſand; upon which again place a lay of maich aş before, and upon this another of ſand, and ſo interchangeably till the pot is full, but finiſhing always with a lay of ſand: then cover it with an earthen cover, and lute with clay the joining, ſo as no air may get entrance. This done put burning coals round the pot, and after it has been kept

kept hot for ſome hours, let it cool of itſelf; ſo your match is prepared, which will burn without ſmoak or offenſive ſmell.

SPORT LVII.

To make Golden-rain for Sky-rockets.

THERE are ſome ſky-rockets, which in falling make little waves in the air, like unto hair half curled, and are therefore called Hairy-Rockets; they end in a ſort of rain of fire, called Golden Rain. 'Tis thus made.

Fill with the compoſition for ſky-rockets gooſe-quills, the feathers being cut off; putting ſome wet Powder in the open end of each, both to keep in the compoſition, and to ſerve for priming: with theſe fill the head of your ſky-rocket, and it will end in a golden rain very agreeable to behold.

Remark. This golden-rain calls to my mind a Pyrotechnical Hail, ſo called from its reſemblance to the natural, which is a quantity of ſmall hard bodies, being either pieces of flint, round ſtones, leaden bullets, or ſquare pieces of iron, incloſed in a catridge of wood, iron, or copper, and is therefore called Catridge or Caſe-ſhot; they are uſed in war, either in open field to diſorder an enemy's army, or in a ſiege to drive them away from a breach or gate to be ſeized, being ſhot either out of a mortar, or a great-gun of a large bore.

SPORT LVIII.

To repreſent with Rockets, ſeveral figures in the air.

IF you take a rocket of the larger ſort, and place round the head of it many ſmall ones, fixing their ſticks all round the large coffin upon the head of your big rocket, which uſes to contain the head works, ordering it ſo, that your ſmall rockets take

fire whilſt the great one is mounting up, you will have the reſemblance of a tree, very delightful to the ſight; whereof the big one will repreſent the trunk, and the little ones the branches.

But if the ſmall rockets take fire when the great one is half turned in the air, they will have the appearance of a comet: and when the large one is altogether turned, ſo that its head paints downwards to the earth, they will exhibit the ſimilitude of a fountain of fire.

If you put on the head of a large rocket many gooſe-quills, the feathers being cut off, filled with ſky-rocket compoſition, as in the preceding ſport; when fired, they will appear to thoſe under them as a fine ſhower of fire; but to thoſe who view them on one ſide, like half curled hair, very delightful to the view.

Finally, with ſerpents tyed to a rocket with pack-thread by the ends which are not fired, leaving two or three inches of the thread between each, you may repreſent at pleaſure ſeveral ſorts of figures moſt entertaining and agreeable to the ſight.

SPORT LIX.

To make Fire-Pots for fireworks of diverſion.

A Pot of fire is a large Coffin filled with Rockets that take fire all together, and are diſcharged from the Pot without hurting it. The bottom of the Pot muſt be covered with Powder-duſt, which being fired by a match that muſt paſs through a hole in the middle of the Pot, will ſet fire to all the Rockets at once.

When there are many Fire-pots, they muſt be covered with ſingle paper, that they may not play all at once; otherways, one when fired might ſet fire to another; and you muſt uſe only a ſingle leaf of paper, that it may not hinder the Rockets to fly out. Pots of Fire are alſo made for war-ſervice.

SPORT

SPORT LX.

To make Shining-balls for diverſion, and for ſervice in war.

FIRST, to make ſhining balls for recreation; to four pounds of ſalt-petre, put ſix pounds of ſulphur, two pounds of crude antimony, four pounds of colophony, and four pounds of coal: or, to two pounds of ſalt-petre, take one pound of ſulphur, as much antimony, two pounds of colophony, as much coal, and one pound of black pitch; melt theſe, being well beaten, in a kettle, or in a glazed earthen pot, and thereinto throw ſuch a quantity of hards of flax or of hemp, as will juſt ſuffice to imbibe all the liquor, of which as it cools make little pellets or round balls, to be covered over with prepared tow, which I taught to make in Sport 56 and after put into ſky-rockets, or balls for eiverſion, as is uſual to be done with fiery ſtars.

Next, to make Shining or Flaming-balls for ſervice in war, to be thrown from a mortar, againſt the enemy, you muſt melt in a kettle, or glazed earthen pot, as above, equal parts of ſulphur, black pitch, roſin, and turpentine, into which dip an iron, or ſtone bullet, ſomewhat lower than the bore of the mortar, and when its ſurface is covered with this matter, rowl it in corn-powder: which done cover it over with calico, and dip it again into the ſame liquor; rolling it after in grain powder; this muſt be reiterated ſeveral times, covering, dipping, and rowling it, till it fills exactly the bore of the mortar or cannon, into which you deſign to put it, remembering ſtill to end your operations with rolling it in grain-powder, that being put into your piece, immediately above the charge of the powder, it may take fire as it is thrown into the air againſt the enemy

enemy, either to annoy them, or to difcover their defigns, which is ufually done in fieges.

Inftead of thefe fhining-balls, red-hot-balls are more frequently ufed for offending the enemy, by burning them, their houfes, or works. Thefe bullets are of iron, and being hated red-hot in a furnace are thus ufed. Your cannon being charged with powder, freed from corns, and pointed fomething upwards, you muft have in readinefs a cylinder of wood fitted exactly to its bore, which you muft put into your gun next the powder, and upon it you muft ram down a wad of wet ftraw, hay, or tow of hemp, or fome fuch moift materials: then putting in your red-hot-ball with a ladle, immedidiately put fire to your gun.

SPORT LXI.

To make a Wheel of Fire-works.

A Wheel of Fire, or Fire-works, is a wheel of light wood, fet round with rockets of a midfize, the head of one regarding the tail of another, that when the firft is fpent, it may fet fire to the next, which makes the wheel turn round its fixed axel-tree without intermiffion, till all the rockets are confumed.

Upon this account it is called a Fire-wheel, and it is alfo called a *Fiery Sun*, becaufe placed horizontally upon a ftake fomewhat large and perpendicular to the horizon, it turns round, and reprefents a fun in night combats, which is very diverting.

You may alfo make fire-wheels which have a fituation perpendicular to the horizon, and turn upon an axis parellel to it, very agreeable to behold. Fire-wheels are likewife ufed to light other works at a diftance, in afcending or defcending upon a ftretched rope, like flying-dragons; and on many

many other occaſions, to the pleaſure of the Spectators.

SPORT LXII.

To make a Balloon, or fiery Foot-ball.

BALLOONS are coffins of a large diameter, ſhot out of a mortar whither one pleaſes, filled commonly with Serpents about the thickneſs of a Ground-rocket, but not ſo long, with two ſmall Fire-links of the ſame length and breadth, which being fired by their priming, burſt the Coffin, this having below a fire-conveyance, at the mouth of which there is a priming of cotton dipt in Powder.

The Coffin is made with a thick wooden Rowler; about which is rowled ſtrong card-paper, glued to keep it from undoing, which being choaked below, a hole is made there for a fire conveyance, filled with a Compoſition more ſlow than that of Ground-rockets, being like to that of Sky-rockets: after this it may be filled with Serpents, and ſometimes with Stars, and then choaked above.

SPORT LXIII.

To make an Ointment excellent for curing all ſorts of burnings.

BOIL, over a gentle fire, in common water, Hogs' Lord, or the fat of freſh Pork, ſkimming it perpetually till no furthe ſcum ariſes; then expoſe it thus melted to cool in the clear open air three or four nights. After this melt the ſame lard or greaſe in an earthen veſſel over a ſlow fire, and ſtrain it through a linen cloth into cold water, and after waſh it well in fair river or fountain water, to take away its ſalt, which will make it become white as ſnow. Finally, being thus purified, put it up in a glazed earthen veſſel, to be kept for uſe upon occaſion.

If

If it falls out, as commonly it happens, that by a burning, bliſters riſe upon the ſkin, they muſt not be cut or broken, till the ointment has been uſed to it for three or four days. You may alſo uſe the following, which you will find to be of great efficacy, and is made of Hogs' Lard melted and mixed with two drams of the water of Nightſhade, and one dram of Oil of Saturn; or with two ounces of Juice of Onions, and one ounce of Oil of Walnuts.

PART III.

Miſcellaneous SPORTS.

SPORT LXIV.

To deſcribe an artificial lantern, by which one may read at night at a great diſtance.

MAKE a lantern in the form of a cylinder or of a ſmall caſk laid on one ſide; put in one of its two ends a concave parabolic glaſs, in order to apply to its focus the flame of a wax-candle, the light of which will reflect to a great diſtance in paſſing through the other end that ought to be open and will appear with ſuch a ſplendor, that by it one may read at night very ſmall letters at a great diſtance, with teleſcopes; and thoſe who ſee the light of the candle at a great diſtance, will take it to be a great fire, which will be ſtill the lighter if the lantern is tinned within, and made in the form of an elypſis.

The

The Magical Lantern.

We likewise make use of such a glass for a magical lantern, so called, because by means of it we can make any thing appear on the white wall of a dark room; such as monsters and fearful apparitions, which the ignorant impute to magic. The light reflected by virtue of this glass passes through a hole in the lantern, in which there is a lens of glass; and between them there is a thin piece of wood containing several glasses painted with monstrous and formidable figures, which they move up and down through a slit in the body of the lantern, and which cast their representation to any opposite wall in the same colours and proportions, but much enlarged.

SPORT LXV.

By the means of two plain looking-glasses to make a face appear under different forms.

HAVING placed one of the two glasses horizontally, raise the other to about right angles over the first; and while the two glasses continue in this posture, if you come up to the perpendicular glass, you will see your face quite deformed and imperfect; for it will appear without forehead, eyes, nose or ears, and nothing will be seen but a mouth and a chin raised bold. Do but incline the glass never so little from the perpendicular, and your face will appear with all its parts excepting the eyes and the forehead. Stoop it a little more, and you will see two noses and four eyes; and then a littl further, and you will see three noses and six eyes. Continue to incline it still a little more, and you will see nothing but two noses, two mouths and two chins; and then a little further again and you will see one nose, and one mouth. At last incline a little further, that is, till the angle of incli-

inclination comes to be 44 degrees, and your face will quite diſappear.

If you incline the two glaſſes the one towards the other, you will ſee your face perfect and intire; and by the different inclinations, you will ſee the repreſentation of your face, upright and inverted alternately, &c.

SPORT LXVI.

To deſcribe a vertical dial upon a pane of glaſs ſo as to denote the hours without a gnomon.

I Once made ſuch a dial for a friend after the following manner.

I took off a pane of glaſs that was ſoldered on the out-ſide to the frame of a window, and calculating the thickneſs of the frame for the gnomon, had the pane glewed on again to the inſide of the frame, allotting to the meridian line a ſituation perpendicular to the horizon, as it ſhould be in vertical dials, and on the outſide, I cauſed to be glewed to to the frame oppoſite to the dial, a ſtrong piece of paper un-oiled, that ſo the rays of the ſun might penetrate it the leſs, and keep the ſurface of the dial darker. Then to diſtinguiſh the hours without a ſtyle, I made a little hole in the paper with a pin, over-againſt the foot of the ſtyle marked upon the dial; and thus the hole repreſenting the tip or end of the ſtyle, and the rays of the ſun paſſing through it, caſt upon the glaſs a ſmall light that pointed out the hours very prettily in the obſcurity of the dial.

SPORT LXVII.

To find in all parts and at all times, the four cardinal points of the world, without ſeeing the ſun, or the ſtars, or making uſe of a compaſs.

THE four cardinal parts of the world, *viz.* the eaſt, the weſt, the ſouth and the north, are

are easily found by a compass, the needle of which being touched with a loadstone, turns always one of its points towards the south and the other towards the north, which is enough to direct us to east and west; for when one sets his face to the north, the east is on his right and the west is on his left hand.

The north is easily distinguished in the night by the stars, particularly by minding the polar star, which is but two degrees distant from the arctic pole: and in the day-time astronomers mark the meridian line upon an horizontal plain, by means of the two points of a shadow marked before and after noon upon the circumference of a circle described from the point of the stylus, the shadow of which is made use of to shew by its extremity mpon that circumference two points equally remote from the meridian.

But without all these helps you may at all times, and in all parts, find out the meridian line, after the following manner.

Take a platter or basin full of water, and when the water is settled and still, put softly into it an iron or steel needle, such as a common sewing needle; and if the needle is dry, and be laid all along upon the surface of the water, it will not sink; but after several turns will stop in the plan of the meridian circle, so that it represents the meridian line; and by consequence one end of it will point to the south, and the other to the north: but without seeing the sun or the stars, it is not easy to know which of the two ends points to the south, and which to the north.

Father Kircher lays down an easy way of knowing south and north. He orders you to cut horizontally a very straight tree growing in the middle of a plain at a distance from any eminence or wall that

may ſhelter it from the wind or the rays of the ſun: in the ſection of that trunk you will find ſeveral curve lines round the ſap which lie cloſer on one ſide than the other:

And, as he ſays, the north lies on that ſide where the lines are moſt contracted, perhaps becauſe the cold ariſing from the north binds up, and the heat from the ſouth ſpreads and rarifies the humours and matter, of which theſe crooked lines are formed. Theſe lines, ſays that author, are as the circumferences of concentrical circles in ebony or Brazil wood.

SPORT LXVIII.

To break with a ſtick another ſtick reſting upon two glaſſes without breaking the glaſſes.

THE ſtick that is to be broken muſt not be very thick, nor yet lean much upon the glaſſes; it ought, as near as poſſible, to be equally thick all over, for the eaſier finding of its center of gravity, which will then be in the middle.

The ſtick being thus qualified, we lay its two ends, which ought to terminate in a point, upon the brim or edge of two glaſſes of equal height, ſo that the ſtick does not lean to one ſide or end more than the other, and the two points ends reſt but lightly upon the edge of each glaſs, to the end that when it bends a little through the violence of the ſtroke, it may eaſily ſlip off, and break at the ſame time. This done, we take another ſtick, and with that give a ſmart blow upon the midling point, which being the centre of gravity will receive all the force of the blow; thus will the ſtick break, and that the more eaſily that the blow is violent, and fall clear of the two glaſſes which remain unbroken, becauſe the ſtick lay but very gently and equally upon the brim of each; for if it reſts more upon one glaſs

than

than the other, it will preſs that one moſt, and ſo may break it.

SPORT LXIX.

To make a lamp fit to carry in one's pocket, that ſhall not go out though you roll it upon the ground.

TO make a lamp that never ſpills its oil, and never goes out in any poſition whatſoever, make faſt the veſſel that contains the oil and the match to an iron or braſs ring, with two ſmall pivots or hinges diametrically oppoſite, that ſo the veſſel may by its weight continue in æquilibrio round the two hinges and turn with freedom within the circle, ſo as to keep always to an horizontal poſition, as in your ſea compaſſes, which have two ſuch circles to keep them horizontally; and in like manner this firſt circle ought to have two other pivots diametrically oppoſite, which enter into another circle of the ſame ſubſtance; and that ſecond circle has two other little hinges inſerted in another concave body that ſurrounds the whole lamp. Thus the lamp with its two circles may turn freely upon its ſix hinges, which give to the lamp when it is turned, ſix different poſitions, *viz.* up and down, forwards and backwards, to the right and left, and which ſerve to keep the lamp in an horizontal poſition, which, being in the middle does always reſt upon its center of gravity, that is, its center of gravity is always in the line of direction, which hinders the oil to ſpill, turn it which way you will.

SPORT LXX.

To take up a boat that is ſunk with a cargo of goods.

IF a boat ſinks in a deep river, you may bring her up again, by getting two other boats, one

empty, and the other deep loaded with ſome heavy ſubſtance, as ſtones, &c. You muſt tie theſe two boats to the boat that is ſunk with two ropes, and extending the rope of the deep laden boat, unload her into the other that is empty; which will raiſe the firſt boat a little, and make it draw along with it the boat that is under water, and at the ſame time make the ſecond boat ſwim ſo much deeper in the water. The ſecond boat being thus loaded, you muſt bend her roap and unload her again into the empty boat, and thereupon ſhe becoming lighter, will riſe and draw the boat under water ſo far further up. Thus you continue to load and unload till you bring the boat even with the water, and then tow her to the ſide.

SPORT LXXI.

To make a boat go of itſelf up a rapid current.

THE more rapid a river is, the eaſier it is to make a boat go of itſelf up againſt the current, by a rope and a wheel with its axeltree that has wings like the wings or ſweeps of a mill-wheel.

Fix the wheel with its axeltree at the place to which you would have the boat conducted, and let its ſweeps be as deep in the water as there is occaſion for turning it round: tie a roap to the boat and to the axeltree of the wheel, which turning with its axeltree by virtue of the rapidity of the water, will wind up the rope on its axeltree, and ſo by the ſucceſſive abreviation of the rope, drag it againſt the current to the place propoſed: which it will reach ſo much the ſooner that the current is rapid, the rapidity quickening the motion of the wheel.

SPORT LXXII.

To know which of two different waters is the lighteſt, without any ſcales.

TAKE a ſolid body, the ſpecific gravity of which is leſs than that of water, dale or fir-wood, for inſtance; and put it into each of the two waters, and reſt aſſured that it will ſink deeper in the lighter than in the heavier water; and ſo by obſerving the difference of the ſinking you will know which is the lighteſt water, and conſequently the wholſomeſt for drinking.

SPORT LXXIII.

To know if a ſuſpicious piece of money is good or bad.

IF it be a piece of ſilver that is not very thick, as a crown or half crown, the goodneſs of which you want to try; take another piece of good ſilver of equal balance with it, and tie both pieces with thread or horſe-hair to the ſcales of an exact balance (to avoid the wetting of the ſcales themſelves) and dip the two pieces thus tied in water; for then if they are of equal goodneſs, that is, of equal purity, they will hang in æquilibrio in the water as well as in the air; but if the piece in queſtion is lighter in the water than the other, it is certainly falſe, that is, there is ſome other metal mixed with it that has leſs ſpecific gravity than ſilver, ſuch as copper; if it is heavier than the other, it is likewiſe bad, as being mixed with a metal of greater ſpecific gravity than ſilver, ſuch as lead.

If the piece propoſed is very thick, ſuch as that crown of gold that Hiero king of Syracuſa ſent to Archimedes to know if the goldſmith had put into it all the 18 pounds of gold that he had given him

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for that end; take a piece of pure gold of equal weight with the crown propoſed, *viz.* 18 pounds, and without taking the trouble of weighing them in water, put them into a veſſel full of water, one after another, and that which drives out moſt water, muſt neceſſarily be mixed with another metal of leſs ſpecific gravity than gold, as taking up more ſpace though of equal weight.

SPORT LXXIV.

To know how the wind ſtands, without ſtirring out of one's chamber.

FIX to the cieling of your room a circle divided into 32 equal parts, with the names of the 32 rumbs or wind-points, the points of north and ſouth being upon the meridian line. The circle or dial thus divided, muſt have a needle or hand movable round its centre, like the hand of a watch or clock; and that hand muſt be fixed to an axeltree that is perpendicular to the horizon, and may move eaſily upon the leaſt wind, by virtue of a fane on its upper end above the roof of the houſe, and then the wind turning the fane, will at the ſame time turn its axeltree, and the hand that is fixed to it, which will accordingly point to the rumb from whence the wind blows.

SPORT LXXV.

When two veſſels or cheſts are like one another, and of equal weight, being filled with different metals, to diſtinguiſh the one from the other.

THIS ſport is eaſily performed, if we conſider that two pieces of different metals of equal weight in air, do not weigh equally in water: becauſe that of greateſt ſpecific gravity takes up a leſſer ſpace in water, it being a certain truth, that any metal weighs leſs in water than in air, by reaſon of

of the water, the room of which it fills. For example, if the water weighs a pound, the metal will weigh in that water a pound lefs than in the air. This gravitation diminifhes more or lefs according as the fpecific gravity of the metal is greater than that of the water.

We will fuppofe then two chefts perfectly like one another, of equal weight in the air, one of which is full of gold, and the other of filver; we weigh them in water, and that which then weighs down the other muft needs be the gold cheft, the fpecific gravity of gold being greater than that of filver, which makes the gold lofe lefs of its gravitation in water than the filver. We know by experience, that gold lofes in water about an eighteenth part only, whereas filver lofes near a tenth part: fo that if each of the two chefts weighs in the air, for example, 180 pounds, the cheft that is full of gold will lofe in the water ten pounds of its weight; and the cheft that is full of filver will lofe eighteen; that is, the cheft full of gold will weigh 170 pounds, and that of filver only 162.

Or, if you will, confider that gold is of a greater fpecific gravity than filver, the cheft full of gold, though fimilar and of equal weight with the other, muft needs have a leffer bulk than the other. And therefore if you dip feperately each of them into a veffel full of water, you may conclude that the cheft which expels lefs water has the leffer bulk, and confequently contains the gold.

SPORT LXXVI.

To meafure the depth of the fea.

TIE a great weight to a very long cord, or rope; and let it fall into the fea till you find it can defcend no farther, which will happen when the weight touches the bottom of the fea, if the

quantity or bulk of water the room of which is taken up by the weight and the rope, weighs leſs than the weight and rope themſelves; for if they weighed more, the weight would ceaſe to deſcend, though it did not touch the bottom of the ſea.

Thus one may be deceived in meaſuring the length of a rope let down into the water, in order to determine the depth of the ſea; and therefore to prevent miſtakes, you had beſt tie to the end of the ſame rope another weight heavier than the former, and if this weight does not ſink the rope deeper than the other did, you may reſt aſſured that the length of the rope is the true depth of the ſea: if it does ſink the rope deeper, you muſt tie a third weight yet heavier, and ſo on till you find two weights of uneqnal gravitation that run juſt the ſame length of the rope, upon which you may conclude that the length of the wet rope is certainly the ſame with the depth of the ſea.

SPORT LXXVII.

To repreſent lightning in a room.

THE room in which you are to repreſent lightening muſt not be large, but quite dark, and ſo very cloſe that the air cannot readily enter it. The room being thus in order, take a baſin into it with ſpirit of wine and camphyr, which muſt boil there till it is all conſumed, and nothing left in the baſin. This will rarify the camphyr, and turn it into a very ſubtile vapour, which will diſperſe itſelf all over the room; inſomuch that if any one enters the room with a lighted flambeau, all the impriſoned vapour will in a moment take fire, and appear as lightening, but without hurting either the room or the ſpectators.

Remark. Camphyr is of a nature ſo proper to retain and keep an unextinguiſhable fire that it will

burn entirely, and that very eaſily upon ice or among ſnow, which it melts, notwithſtanding their coldneſs; and if it be reduced to powder and thrown upon the ſurface of any ſtill water, and then lighted, it will produce a very pleaſant ſort of fire, for the water will appear all fire and flame; the reaſon of which I take to be, becauſe the camphyr is of a fat nature which reſiſts water, and of a light and fiery ſubſtance, which the fire graſps ſo keenly that it is impoſſible for this ſubſtance to diſengage itſelf when once it is entangled.

SPORT LXXVIII.

To melt at the flame of a lamp a ball of lead in paper, without burning the paper.

TAKE a very round and ſmooth leaden ball, wrap it up in white paper, that is not rumpled, but clings equally about the ball without wrinkles, at leaſt as far as is poſſible; hold the ball thus wrapped up over the flame of a lamp or flambeau, and it will grow hot by degrees, and in a little time melt and fall down in drops through a hole in the paper without burning it.

SPORT LXXIX.

To repreſent an Iris or rainbow in a room.

EVERY one knows that the rainbow is a great arch of a circle that appears all on a ſudden in the clouds before or after the rain, towards that part of the air that is oppoſite to the ſun, by virtue of the reſolution of the cloud into rain; this arch is adorned with ſeveral different colours, of which the principal are five in number, namely red which is outermoſt, yellow, green, blue, and violet and purple, which is interior.

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This Iris feldom appears alone, and is called the firft and the principal rainbow, to diftinguifh it from another that commonly appears along with it, and for that reafon is called the fecond rainbow, the colours of which are not fo lively as thofe of the firft, though they are difpofed after the fame manner, but in a contrary order, upon which account a great many take it for a reflection of the firft.

If you want to reprefent, at one time two fuch Iris's in your room, put water into your mouth and ftep to the window (upon which the fun is fuppofed to fhine) then turn your back to the fun, and your face to the dark part of the room; and blow the water which is in your mouth, making it fpurt out with violence, into little drops or atoms; and among thefe little attoms or vapours, you will fee by the rays of the fun, two rainbows refembling the two that appear in the heavens in rainy weather.

Oftentimes we fee rainbows in waterworks or fpouts, when we ftand between the fun and the fountain, efpecially when the wind blows hard, for then it difperfes and divides the water into little drops. Which is full evidence, that the rainbow, which the philofophers admire as much as the ignorant people do thunder, is formed by the reflection and refraction of the rays of the fun darted againft feveral little drops of water, that fall from the clouds in time of rain.

A rainbow may likewife be very eafily reprefented, in a room with a window that the fun fhines upon, by a triangular prifm expofed to the rays of the fun, which in paffing through the glafs, will by their different reflections and refractions produce upon the wall or ceiling of the room, a very agreeable Iris, or at leaft a texture of feveral different colours refembling thofe of the rainbow; and the further the ceiling

ceiling or wall is diſtant, and the more it is dark, the colours will appear the more charming and lively. You may likewiſe imitate the colours of the rainbow by expoſing to the ſun a ſphere of cryſtal or glaſs, or a glaſs full of clean water.

SPORT LXXX.

To make a conſort of muſick of ſeveral parts with only one voice.

THE ſound conveyed diſtinctly to the ear, by remote bodies, againſt which the air is driven by the voice of an animal or otherwiſe, and then reflected, is what we call an echo; which is ſometimes double, triple, *&c.* when the voice is ſtrong enough to make ſeveral bodies, at different diſtances, beat back at ſeveral times the parts of the air to our ears, ſo that one echo is no ſooner ended than another begins.

Though moſt eccho's make us hear only the laſt words of the voice, becauſe the air, though ſtrongly impreſſed, has not the ſame force at the end that it had at the beginning; yet it may be ſo contrived as to make a conſort of muſic of ſeveral parts, that is, a conſort of ſeveral ſongs tuned together, by only one voice or one inſtrument, to the ſound of which the eccho anſwers.

For if the echo anſwers only once to the voice or the ſound of the inſtrument, he who ſings or plays may make a Duo, that is, a muſic of two parts; and again a Trio or muſic of three parts, if the echo anſwers twice. But indeed he muſt be an expert muſician, and one that is well verſed in varying the tune and the note.

Thus commencing, for example at Ut, he may begin Sol a little before the echo anſwers, ſo as to finiſh the pronunciation of Sol by the time that the eccho has completed its anſwer, and then he will

have

have a fifth, which is a perfect consonance in music; and in like manner, if at the same time with the echo's answering to the second note Sol, or a little before, he repeats it upon a higher or lower note, he will make a diapason or eighth, which is perfect harmony in music. And so on, if he has a mind to continue the chace with the echo, and sing alone the two parts.

To this purpose we see by experience in several churches, when they are singing, that there seems to be many more parts in the chorus than there really are, the quantity of echo's making the air to resound on all sides, and so multiplying the voice and redoubling the chorus.

SPORT LXXXI.

To make a string of a vial shake without touching it.

CHOOSE at pleasure three strings in a viol, or any other instrument of that sort, without any intermediating string, and tune the first and the third to the same note, without touching that in the middle; then strike one of the two strings thus tuned pretty hard with a bow, and you will find that when it shakes the other will tremble sensibly and visibly, and the middle string, though nearer, shall stir no manner of way.

This sport may likewise be resolved by two stringed instruments of the same sort, as two viols, two lutes, two harps, two spinets, &c. by putting the two in the same tune, and then placing them at a convenient distance, and in a proper position; for one of the two instruments being touched with a midling force, will move the other, that is, the strings of the other, which are supposed to be in unison, will produce such another harmony, especially if the strings in one and the other instrument

are

are equally long and equally thick. For this I can aſſign no other reaſon but experience.

SPORT LXXXII.

To make a deaf man hear the ſound of a muſical inſtrument.

IT muſt be a ſtringed inſtrument, with a neck of ſome length, as a lute, a guitarre, or the like; and before you begin to play, you muſt by ſigns direct the deaf man to take hold with his teeth of the end of the neck of the inſtrument; for then if one ſtrikes the ſtrings with a bow one after another, the ſound will enter the deaf man's mouth, and be conveyed to the organ of hearing through the hole in the palate: and thus the deaf man will hear with a great deal of pleaſure the ſound of the inſtrument, as has been ſeveral times experienced. Nay, thoſe who are not deaf may make the experiment upon themſelves, by ſtopping their ears ſo as not to hear the inſtrument, and then holding the end of the inſtrument in their teeth while another touches the ſtrings.

SPORT LXXXIII.

To make an egg enter a vial without breaking.

LET the neck of the vial be never ſo ſtrait, an egg will go into it without breaking, if it be firſt ſteeped in very ſtrong vinegar, for in proceſs of time the vinegar does ſo ſoften it, that the ſhell will bend and extend lengthways without breaking. And when it is in, cold water thrown upon it will recover its primitive hardneſs, and, as Cardan ſays, its primitive figure.

SPORT LXXXIV.

To make an egg mount up of itſelf.

MAKE a little hole in the ſhell of the egg, and ſo take out the yelk and the white, and fill

fill the egg-ſhell with dew, then ſtop up the hole and expoſe it to the rays of the ſun at noon day; for then the dew not being able to bear the light, nor too great heat, will rife up with the egg-ſhell, eſpecially if it leans againſt a little ſtick or piece of wood, that ſlopes never ſo little, and if the hole is well ſtopped. May dew is ſaid to be beſt; and it is obſerved by the farmers, that the more May abounds in dew, the more plentifully does the earth bring forth: for dew being a ſubtile vapour produced in the morning by a weak heat, and preſerved by a moderate cold, it is very well diſpoſed for the reception of celeſtial virtues; and when it inſinuates itſelf into vegetables, it communicates to them the virtues it retains; and hence it comes that plants moiſtened with it thrive better, than when they are nouriſhed with ſpring, well, or river water.

SPORT LXXXV.

To make water freeze any time in a hot room.

FILL a vial with warm water, the neck of which is ſomewhat narrow, and having ſtopped it cloſe, put it in a veſſel full of ſnow mixed with common ſalt and ſaltpetre, ſo as to leave the vial covered all over with ſnow: and in a little time the water will be quite frozen, though in the ſummer time, and in a very hot room.

If you throw cold water with ſnow upon a table, and upon the ſnow ſet a platter full of ſnow with a ſufficient quantity of ſalt and ſaltpetre pounded; the ſalt and the ſaltpetre will make the ſnow ſo cold, that in a little time the water under the platter will be turned to ice, and make the platter ſtick ſo faſt to the table, that you cannot move it without ſome difficulty.

Remark. The ſaltpetre and ſal-armoniac are likewiſe poſſeſſed of the virtue of making water ſo

extremely

extremely cold, that if you put a ſufficient quantity of them in common water, it will become ſo cold that your teeth can ſcarce bear it. They might therefore be very uſefully employed in ſummer for cooling wine or any other liquor, by ſetting the wine bottles in water thus refrigerated.

If you diſſolve a pound of nitre in a pail of water, the water will be exceſſive cold, and ſo very proper for the uſes above-mentioned. It is well known that wine is cooled with ice; and in regard ice cannot always be had in ſummer, I ſhall preſcribe a way of making it.

To make Ice in ſummer.

To make ice in ſummer, put two ounces of refined ſaltpetre, and half an ounce of florentine Oris, into an earthen bottle filled with boiling water, ſtop the bottle cloſe, and convey it forthwith into a very deep well, and there let it ſteep in the well-water for two or three hours, at the end of which you will find the water in the bottle all ice: ſo you have nothing to do but to break your bottle and take out your ice.

SPORT LXXXVI.

To kindle a fire by the ſun-beams.

THIS Sport may be performed either by refraction in uſing lenticular glaſſes thicker in the middle than in the ſides, called burning-glaſſes, through which when the rays paſs they refract and unite into one point called the Focus, at which you may light a match or any other combuſtible matter: or elſe by reflexion, in uſing a concave looking-glaſs of metal well poliſhed in its concavity, which may be either ſpherical or parabolic, and is likewiſe called a burning-glaſs, but much better than the former ſort; for by it you may in a moment ſet fire to a piece of wood, and in a ſhort

ſhort time melt lead, and even iron, and vitrify ſtone.

SPORT LXXXVII.

To make a fowl roaſting at the fire, turn round of itſelf with the ſpit.

TAKE a wren and ſpit it upon a hazel ſtick, and lay it down before the fire, the two ends of the hazle ſtick being ſupported by ſomething that is firm ; and you will ſee with admiration the ſpit and the bird turn by little and little without diſcontinuing, till it is quite roaſted. This experiment was firſt found out by Cardinal Palotti at Rome, who ſhewed it father Kircher, in order to know the phyſical cauſe of it ; which to my mind is eaſily diſcovered, for the hazel wood is compoſed of ſeveral long and porous fibres, into which the heat inſinuates itſelf, and ſo makes it turn round when the wood is hung right.

SPORT LXXXVIII.

To make an egg ſtand on its ſmalleſt end, without falling, upon a ſmooth plain ſuch as glaſs.

PLACE a looking-glaſs quite level, or horizontally, without inclining to either ſide; toſs the egg with your hand till the yelk burſts, and the matter of it is equally diſperſed through all the parts of the white, ſo that the white and the yelk make but one body. Then ſet the end of the egg upon the horizontal plain, holding it till it is upright, and then it will continue in that ſituation without falling, by reaſon of the æquilibrium made on all ſides by the parts of the yelk equally mixed with the white, ſo that the center of gravity in the egg continues in the line of direction.

SPORT

SPORT LXXXIX.

To make a piece of gold or ſilver diſappear, without altering the poſition of the eye or the piece, or the intervention of any thing.

PUT the piece of gold in a porringer full of water, or a veſſel that is broader than it is deep, and let the eye be in ſuch a poſition, as juſt barely to ſee the piece at the bottom over the brim of the veſſel; then take out the water, and though the porringer continues in the ſame poſition as well as the eye, the piece which appeared before by virtue of the refraction made in the water, will then be covered from the ſight by the ſides of the porrenger.

SPORT XC.

To make a loaf dance while it is baking in the oven.

PUT into the dough a nutſhell filled with live ſulphur, ſaltpetre and quickſilver, and ſtopped cloſe; as ſoon as the heat comes to it, the bread will dance in the oven; which is occaſioned by the nature of quickſilver, for it can bear no heat without being in a continual motion. Thus, by the means of quick-ſilver put into a pot where peaſe are to be boiled, all the peaſe will leap out of the pot as ſoon as the water begins to heat. In like manner quickſilver put into hot bread, will make it dance up and down the table.

SPORT XCI.

To ſee in a dark room what paſſes abroad.

MAKE your room ſo cloſe and dark, that the light can come in no where but through a little hole left in a window upon which the ſun ſhines; over againſt this hole, at a reaſonable diſtance from it, place ſome white paper, or a piece

 of

of linen; and you will ſee every thing that paſſes by the outſide of the window appear on the paper or linen, only their figures are inverted.

SPORT XCII.

To hold a glaſs full of water, with the mouth down, ſo as that the water ſhall not run out.

TAKE a glaſs full of water, cover it with a cup that is a little hollow, inverting the cup upon the glaſs; hold the cup firm in this poſition with one hand, and the glaſs with the other, then with a jerk turn the glaſs and the cup upſide-down, and ſo the cup will ſtand upright, and the glaſs will be inverted, reſting its mouth upon the interior bottom of the cup. This done, you will find that part of the water contained in the glaſs will run out by the void ſpace between the bottom of the cup and the brim of the glaſs; and when that ſpace is filled, ſo that the water in it reaches the brim of the glaſs, all paſſage being then denied to the air, ſo that it cannot enter the glaſs, nor ſucceed in the room of the water, the water remaining in the glaſs will not fall lower, but continue ſuſpended in the glaſs.

If you would have a little more water deſcend into the cup, you muſt with a pipe or otherwiſe draw the water out of the cup, to give paſſage to the air in the glaſs; upon which part of the water will fall into the glaſs till it has ſtopped up the paſſage of the air afreſh, in which caſe no more will come down; or, without ſucking out the water in the cup, you may incline the cup and glaſs ſo that the water in the cup ſhall quit one ſide of the brim of the glaſs, and ſo give paſſage to the air, which will then ſuffer the water in the glaſs to deſcend till the paſſage is ſtopped again.

This

This Sport may likewiſe be performed by covering the brim of the glaſs that is full of water, with a leaf of ſtrong paper, and then turn the glaſs, as above; and without holding your hand any longer upon the paper, you will find it as it were glewed for ſome time to the brim of the glaſs, and during that time the water will be kept in the glaſs.

SPORT XCIII.

To make new wine keep its ſweetneſs for ſeveral years.

MR. Lentin informs us, that if you let new wine heat by itſelf, it loſes in a little time all its ſweetneſs, eſpecially if the caſks are left open; but if you boil it upon a fire immediately after the grapes are preſſed, moſt of the volatile principles of the ſweetneſs concentrate, and link themſelves with the more fixed parts of the wine, which preſerves its ſweetneſs for ſeveral years.

Remark. A ſweet and new wine may preſerve its ſweetneſs at leaſt a whole year, if you pitch the caſk well both within and on the outſide, to hinder the water to penetrate into it, and ſo ſpoil the wine, which ought to be put into it before it boils; and keep the caſk well ſtopped in a ciſtern of water, ſo as to be covered all over for a month or thirty days; and then take out the caſk and place it in a cellar.

In the year 1692, I had a caſk full of Burgundy wine brought me in the ſummer to Paris by water, which immediately upon its arrival was clapped into my cellar, and after a few days ſtanding, I found it boiling as if it had been quite new, and that it had reaſſumed its former ſweetneſs, which continued about a month, and after that it proved extraordinary good wine. Some tell you that a

piece of cheeſe or pumice-ſtone thrown into the caſk, will break the violence of fermenting wine.

To recover the ſweetneſs of new wine.

When the new wine has loſt its ſweetneſs, it may be recovered by caſking it up immediately, and putting in the bottom of the caſk half a pound of muſtard-ſeed, leſs or more, according to the ſize of the caſk.

SPORT XCIV.

To know when there is water in wine, and to ſeperate it from the wine.

IF the wine is neither ſweet nor new, but fine and clear of its lee, you may know, (according to Porta and father Schott) whether it is mixed with water or not, by throwing into it apples or pears, for if the wine is unmixed they will ſink to the bottom, if it is mixed they will ſwim above, becauſe the ſpecific gravity of water is greater than that of wine.

Some order wild apples or pears, and if theſe cannot be had, ripe apples or pears. Others make uſe of an egg, and alledge, that when the wine is pure the egg falls ſwiftly to the bottom, but if it is mixed with water, the egg deſcends more ſlowly, the water having, by virtue of its gravity more force to bear up the egg than the wine has.

Now the contrary will happen, if the wine be ſweet and new, that is, when ſuch wine is unmixed, the egg will deſcend ſlower than when it is mixed; by reaſon that new wine unmixed is by virtue of its lee heavier than water, and conſequently becomes lighter, by the addition of water.

When you have diſcovered that the wine is mixed, you may ſeperate the water by a dry bulruſh, according to Mizauld; for the ruſh being a plant that grows and thrives in watry marſhy places, if it be

dried, and one end of it put into mixed wine, the water will insinuate itself into the rush, and so the wine will be left alone. By the same reason, the rush may serve to discover whether the wine is mixed with water or not.

Remark. On the other hand, some pretend you may separate the wine from the water, by putting in a long narrow piece of linen, woollen or cotton cloth, one end of which hangs out of the vessel, as if the wine being lighter would rise and flow out upon the cloth while the water stays behind; but this and several other ways for the same purpose, are disapproved by other authors.

To pour water into wine without mixing.

You may pour wine upon water, without mixing, if you put a toast of bread upon the water in a glass, and while this toast swims above the water, pour in the wine very softly; for then you will see the water remain unmixed at the bottom of the glass without any alteration in its colour.

To know if milk is mixed with water.

Here by the bye I shall shew you the way of knowing when water is mixed with milk; put a little stick into the milk, then pull it out, and let a drop of the milk fall from it upon the nail of your thumb; and if the milk is pure, the drop being thick will stand for some time upon your nail; but if it is deluted with water it will run off immediately.

To turn water seemingly into wine.

You may turn water into wine in appearance, by setting a vial full of water in a cask full of wine, turning the mouth of the vial downwards; for then the water will run out, and the vial will be filled with wine; which the ignorant will take to be a turning water into wine.

SPORT XCV.

To make a metallic body ſwim above water.

THOUGH the ſpecific gravity of water is inferior to that of metals, and conſequently water is uncapable, abſolutely ſpeaking, to bear up a metallic body, ſuch as a ball of lead, yet this ball may be flatted and beat out to a very thin plate, which when very dry and put ſoftly upon ſtill water, will ſwim upon it without ſinking, by virtue of its dryneſs. Thus we ſee a ſteel needle will ſwim upon the water, when it is dry, and laid ſoftly lengthways upon the ſurfacce of ſtill water.

But if you would have a metallic body to ſwim neceſſarily upon water, you muſt reduce it to a very thin plate, and that concave like a kettle, in which caſe the air it contains weighs leſs than the water, whoſe room it poſſeſſes. It is by this contrivance that coppor boats or pontons are made for paſſing whole armies over rivers without any danger.

Remark. If you put tis concave metallic veſſel upon the water, with its mouth perpendicularly down, it will ſtill ſwim, by reaſon that the air contained in its cavity finds no exit; inſomuch that if you puſh it under water and hold it there by force, the detained air will keep the bottom from being wet on the inſide. And by the ſame reaſon you may have a burning coal in the bottom, and find it not extinguiſhed when you take it out of the water, provided you do not hold it long under water, for fire ſtands in need of air to keep it in.

SPORT XCVI.

To make Aquafortis put up cloſe in a bottle boil without fire.

PUT a ſmall quantity of Aquafortis, and of the filings of braſs in a bottle, and you will

ſee ſo great an ebullition, that the bottle will appear quite full, and be ſo hot that you cannot touch it without burning yourſelf.

Remark. In like manner if you mix oil of tartar and oil of vitriol together, you will preſently ſee a very great ebullition with a ſenſible heat, though neither of theſe liquors is compoſed of any combuſtible matter.

Of Aquafortis and Aqua Regia.

Aqafortis is ſo called with reſpect to its ſtrength in diſſolving almoſt all metals and minerals. It is commonly a diſtillation from ſaltpetre and vitrol or green copperas; and it is yet better, if it be a diſtillation from ſaltpetre and roach allum. It diſſolves all metals but gold; but is rendered capable of the diſſolution of gold by diſſolving ſal armoniac or ſea-ſalt in it, after which it aſſumes the name of Aqua Regia.

Of Sal Armoniac, Roch Allum, and Saltpetre.

To avoid all obſcurity of terms; I ſhall here acquaint you by the bye, that Sal Armoniac is a compoſition of bay-ſalt, chimney-ſoot, and the urine of animals: That roch allum is a mineral earthy ſharp ſalt, filled with an acid ſpirit, which is oftentimes found condenſated in the veins of the earth, or is taken from aluminous ſprings by evaporation; or is found among mineral ſtones, and diſengaged from them by diſſolution in water and evaporation. And in fine, that ſaltpetre is a ſalt that is partly ſulphureous and volatile, and partly terreſtrial; it is found in the dark cavernous places of the earth, and likewiſe in ſtables, by reaſon of the great quantity of volatile ſalt in the urine and excrements of animals, which joins in with the ſalt of the earth by the continual action of the air.

Vitriol.

The oil of Vitriol (mentioned above) is a cauſtic

 oil

oil distilled by a strong reverberating fire from Vitriol. Now, Vitriol is a mineral salt, approaching to the nature of roach-allum, which is found chrystalised in the earth of such mines as abound in metals, which gives us to know that it contains in it some metallic substance, and particularly iron or copper. When it is loaded with copper, if you rub it against iron, it will stain it with a copper colour. But it is best for all manner of preparations when it partakes most of iron.

Tartar.

The oil of Tartar, (mentioned above) is distilled from Tartar along with the spirit, from which it is separated by a funnel lined with brown paper. Tartar itself is an earthy incorruptible substance, formed like a reddish crust round the inside of wine-casks, which thickens and congeals to the hardness of a stone, and is separated from the pure parts of the wine, by the action of the fermentative spirit.

SPORT XCVII.

To make fulminating or thundering powder.

TAKE three parts of saltpetre, two parts of salt of Tartar, and one part of sulphur, pounded and mixed together; heat in a spoon 60 grains of this composition, and it will fly away with a fearful noise like thunder, and as loud as a cannon, breaking through the spoon and every thing underneath it, for it exerts itself downwards, contrary to the nature of gunpowder, which exerts itself upwards.

Salt of Tartar.

The Salt of Tartar, here used, is only a solution in water of the black substance that remains after the distillation of the oil of Tartar, and an evaporation of that solution to a dry salt, which must be kept very close, least the moisture of the air should melt it.

SPORT XCVIII.

To make the Aurum Fulminans, or thundering gold.

PUT into a matraſs, upon hot ſand, the filings of fine gold, with a tripple quantity of Aqua Regia, which will diſſolve the gold: mix this ſolution with a ſextuple quantity of ſpring water, and then pour upon it, drop by drop, the oil of Tartar or volatile ſpirit of Sal Armoniac, till the ebullition ceaſes, and the corroſion of the Aqua Regia is over, for then the powder will precipitate to the bottom, which may be dulcified with warm water, and dried with a very ſlow fire.

This powder is much ſtronger than the laſt deſcribed; for if you ſet fire to 20 grains of it, it will act with more violence, and have a louder crack, than half a pound of gunpowder, and two grains of it kindled at a candle have a ſtronger report than a muſket ſhot.

SPORT XCIX.

To make the Sympathetic Powder.

THE Sympathetic powder is nothing elſe but the Roman Vitriol calcined and reduced to a white light powder, which is ſaid to cure wounds at a diſtance, by being put upon a linen cloth dipped in the wounded perſon's blood, or upon a ſword whereon is the blood or pus that comes out of the wound. This cloth or ſword is wrapped up in a white linen cloth, which is opened every day, in order to ſtrew ſome freſh powder upon the blood or pus of the wound. This courſe they continue till the wound is perfectly cured, which happens the ſooner, if the cloth upon which is the blood and the powder, is kept in a place that is neither too hot, nor too cold, nor too moiſt. Nay, it is

neceſ-

neceſſary ſometimes to ſhift the cloth from place to place, according to the different diſpoſitions of the wound, by putting it, for example, in a cold place, when the patient finds an exceſſive heat in the wound.

To calcine the vitriol for the ſympathetic powder, take ſome Roman vitriol, when the ſun is in the ſign of Leo, or in the month of July, diſſolve it in rain water, and filtrate the water through ſinking pater. Then let the water evaporate upon a gentle fire, and you will find at the bottom the vitriol in little hard ſtones of a fire-green colour. Spread theſe ſtones carefully, and expoſe them to the rays of the ſun, ſtirring them often (with a wooden ſpatula; not an iron ſpatula, becauſe the ſpirits of the vitriol are ready to join in with iron, which would rob the ſympathetic powder of its volatile ſpirits, in which all its virtue conſiſts) that the ſtones may be the better penetrated by the ſun, and calcined and reduced to a powder, which will be as white as ſnow. And to render the ſubſtance of the vitriol more pure and homogenous, the diſſolution, filtration, coagulation and calcination ought to be repeated three times.

This wonderful powder muſt be carefully kept in a vial cloſe ſtopped, and in a dry place, for the leaſt moiſture of the air may turn it to vitriol again, and ſo make it loſe its ſympathetic virtue.

We are told that this powder ſtops all bleedings, and mitigates very much all ſorts of pains in any part of the body, particularly the tooth-ach; and that, by application not to the part affected, but to the blood taken from it, and covered up in a linen cloth as above.

Remark. The chymiſts have another calcination of vitriol called Colcothar, which being put into the noſe, ſtops a bleeding at noſe, and provokes

to ſneeze; being of ſovereign uſe in rouzing the ſenſes, wherefore it is given in lethargies. It is alſo ſucceſsfully uſed for drying up wounds and ulcers. This colcothar is only the vitriol kept melted upon a fire till all its humidity is evaporated, and it is reduced to a hard reddiſh brown maſs, whereby it is rendered fit for the cure of the aforeſaid maladies, and many others not here to be mentioned.

SPORT C.

Of the magnetical cure of diſeaſes by tranſplantation.

THE magnetic cure by tranſplantation, is, that which is performed by communicating the diſeaſe to ſome beaſt, tree, or herb; and, as ſome will have it, is founded upon the efflux of the morbific particles, which paſs by inſenſible tranſpiration out of the body of the patient into another animal or plant.

Froman informs us, that a young ſtudent got rid of a malignant fever by giving it to a dog that lay in the bed with him, and died of it: which if true, muſt needs proceed from the inſenſible tranſpiration of the ſubtile matter, that thereupon entered the pores of the dog.

Thomas Bartholin ſays, his uncle was cured of a violent cholic by applying a dog to his belly, which was thereupon ſeized with it; and that his maidſervant was cured of the tooth-ach by clapping the ſame dog to her cheek, and when the dog was gone from her he howled and made ſuch motions, as gave them to know he had the maid's toothach.

Hoffman ſpeaks of a man cured of the gout by a dog lying in the bed with him, who thereupon was ſeized with it. And frequently after the dog had

fits

fits of the gout, as his maſter had uſed to have before. However this be, certain it is, that dogs are often ſubject to the gout, without any infection from men; and this, and the other ſtories of tranſplantation are not here offered for concluſive proofs, but by way of recreation.

Monſieur de Valemont, who ſeems inclinable to believe tranſplantation of diſeaſes, ſays, it is done not only by inſenſible tranſpiration, but likewiſe by ſweat, by urine, by the blood, by the hair, or by taking up what falls from the ſkin, upon a ſtrong friction. For this he brings ſeveral inſtances, and particularly that which follows.

A perſon of quality in England uſed to cure the jaundice at a great diſtance from the patient, by mixing the aſhes of aſh-wood with the patient's urine; and making of that compoſition three, or ſeven, or nine little balls, with a hole in each of them, in which he put a leaf of ſaffron, and then filled it up with the ſame urine. This done he hung theſe balls in a private place where no body could touch them; and from that time the diſeaſe began to abate.

The great virtues of the Aſh tree.

Remark. The Aſh, which is a common tree all over Europe, has merited the appellation of the Vulnerary wood, by reaſon of its peculiar property in curing ſeveral diſeaſes, and above all wounds and ulcers. Not to mention the almoſt incredible virtues aſcribed to it, it is ſaid to ſtop bleeding at noſe, if the face be but rubbed with the wood, and then waſhed with fair water, and if the patient holds in the hand of that ſide where the bleeding is, a piece of the wood till it heats his hand.

SPORT

SPORT CI.

To ſtop a bleeding at noſe, or at any other part of the body.

FATHER Schott the Jeſuit ſays, that to ſtop a bleeding at the noſe, you need only to hold to the noſe the dung of an aſs very hot, wrapped up in an handkerchief, upon the plea that the ſmell will preſently ſtop it. Wecher did the ſame with hogs dung, very hot done up in fine taffata, and put into the noſe.

I have ſeveral times experienced, that a piece of red coral held in the mouth, will ſtop a bleeding at the noſe. Some tell you that the conſtriction of the thumb of the ſide of the noſtril that bleeds, will do the buſineſs.

To ſtop the bleeding of a wound, take a linen cloth in the ſpring when the frogs lay their eggs in the water, and waſh it in that water till it is well impregnated with the frogs eggs; then dry it at the ſun; and after repeating this impregnation and deſiccation three or four times, keep the cloth to be applied to the wound twice in the form of a cataplaſm. We are told the ſecond application will do.

SPORT CII.

To prepare an ointment that will cure a wound at a diſtance.

THE oinment mentioned by Paracelſus is prepared thus, according to Goclenius. Take of the Uſnea or moſs of the ſcull of a man that was hanged, two ounces; mummy, human blood, of each half an ounce; earth-worms waſhed in water or wine, and dried, two ounces and a half; human fat, two ounces; the fat of a wild boar, and the fat of a bear, of each half an ounce; oil of linſeed and oil of turpentine, of each two drams.

John

John Baptist Porta prescribes it a little otherwise by throwing in some bole armeniac, and leaving out the earth-worms, and the bears and boars fat. But let the composition be which it will, it must be well mixed and beat in a mortar, and kept in a long narrow vial. Some say it should be made when the sun is in Libra. The way of using it is this.

Put into the ointment the weapon or instrument that gave the wound, and leave it there; then let the patient wash his wound every morning with his own urine. and apply nothing else to it; after it is well washed and cleansed, let him tie it up tight with a clean white linen cloth, and he will find it will heal without any pain.

Monsieur Vallemont says, if you cannot get the instrument with which the wound was given, you may take another, which if gently conveyed into the wound, and impregnated with the blood and animal spirits residing there, will have the same effect. He adds, that if you want a speedy cure, you must anoint the insttument often, otherwise you may let it lie a day or two without touching it.

The effect of this unguent he imputes to the subtle particles, which are these little agents that disengage themselves from the most spirituous and transpirable ingredients of which this unguent is composed.

To add to the credibility of its operation, he quots father Lana, who observed that when the vines in France were in flower, the wines in Germany, though at a great distance, suffer an efferve-scene; which he explained by the effluviums of the subtile matter, making these to reach as far as the stars, and alledging that if the attoms, which transpire from the terrestrial globe, were not carried

to

to the ſtars, and ſent back from the ſtars to the earth by a perpetual flux and reflux, there would be no phyſical commerce between the heavens and the earth.

SPORT CIII.

To pierce the head of a pullet with a needle without killing it.

THIS is a very eaſy Sport, for there is a place in the middle of a pullet's head, that may be pierced without hurting the Cerebellum. But the needle muſt not be kept in above a quarter of an hour.

SPORT CIV.

To make handſome faces appear pale and hideous in a dark room.

BURN ſome brandy and common ſalt in a glaſs, then put out the fire and all the lights in the room; and the particles of the ſalt and brandy evaporating into the air ſhut up in the room, will make the faces of the people in the room appear through that air hideous, and frightful.

I intimated above, that if, inſtead of brandy, you take good ſpirits of wine mixed with camphyr in a glazed earthen pan put upon hot burning coals, he that enters the room with a lighted candle will be agreeably ſurpriſed; for the candle ſetting fire to the particles of the ſpirit and the camphyr, with which the air is repleniſhed, that air will ſeem to be all in a fire, and the perſon will ſee himſelf in the midſt of flames without being burned.

SPORT CV.

To lift a mortar of ten pounds weight with a wine glaſs.

TURN the mortar up-ſide down, rub the bottom very ſmooth, ſo that there be not the leaſt

leaſt unevenneſs; make a circle the bigneſs of the rim of the glaſs with clay uſed by diſtillers; then take a lighted paper, put it into the glaſs, turn it down on the mortar, fixing it to the circle of clay and luting it quickly with the ſame, ſo as not the leaſt air may enter: when the paper is burned out, and the glaſs is cool, proceed to the experiment and you will find it anſwer.

SPORT CVI.

To make an egg move upon a table.

TAKE an egg, make a ſmall hole at each end, and after you have blown out the contents through theſe holes put a live leech into it, then ſtop up the holes with wax: the egg thus prepared being ſet upon the table, place ſome freſh water at a little diſtance, which the leech immediately ſmelling will make towards, and give a motion to the egg, to the agreeable ſurprize of the ſpectators.

SPORT CVII.

To make an artificial flame.

ANY of the following oils, mixed with compound ſpirits of Nitre, burſt into a flame, *viz.* oil of Carroways, Cloves, Saſſafras, Guiacum, Box, Camphyr, Pepper, Hartſhorn, Blood, and many other oils.

SPORT CVIII.

To make an artificial earthquake.

TO twenty pounds of iron filings add as many pounds of ſulphur; mix, work, and temper the whole together with a little water, ſo as to form a maſs, half moiſt and half dry: this being buried three or four foot under ground, in ſix or ſeven hours time will have a prodigious effect; the earth

will

will begin to tremble, crack and ſmoke, and fire and flame burſt through.

SPORT CIX.

To prepare a lamp that will make a company look of any colour you pleaſe.

IF you would have a green aſſembly, order a lamp to be made of green tranſparent glaſs, put green oil and a green wick in it, and the company will appear green. If yon would have a black company, you muſt do the ſame in black, and ſo in any colour.

SPORT CX.

To bake an egg upon your head.

TAKE a loaf of bread juſt from the oven, cut a hole in it, put the egg in this hole and cover it with the bread cut out, then wrap the loaf in a linen cloth and hold it upon your head for ſome time, the egg will be ſoon done enough.

SPORT CXI.

To make flames iſſue from an egg.

BLOW out the contents of an egg and dry the ſhell in the ſun, then fill it with brimſtone, ſaltpetre, and unquenched lime; cloſe the holes of the egg through which you blew the contents, then throw it into water and it will flame.

SPORT CXII.

To make a ring leap about.

PREPARE a ring made of copper, hollow, fill it with quickſilver and ſtop cloſe the hole again that the quickſilver may not find any vent. This ring put upon an iron plate made very hot with coals underneath, will ſkip about briſkly.

SPORT CXIII.

Of a deceitful bowl, to play withal.

MAKE a hole in one ſide of the bowl, and caſt melted lead therein, and then ſtop up the hole cloſe, that the knavery or deceit be not perceived; you will have pleaſure to ſee that notwithſtanding the bowl is caſt directly to the play, how it will turn away ſideways; for that on that part of the bowl which is heavier upon one ſide than the other, it never will go truly right, if artificially it be not corrected, which will hazard the game to thoſe who know it not; but if it be known that the leady ſide in rolling be always under or above, it may go indifferently right; if otherwiſe, the weight will carry it always ſideways.

SPORT CXIV.

To contrive a veſſel which keeps its liquor when filled to a certain height, but loſes or ſpills it all when filled a little fuller with the ſame liquor.

TAKE a glaſs and run through the middle of it a ſmall bended pipe or crane, open at the end next the bottom of the glaſs, and likewiſe at the other end, which muſt be lower than the bottom of the glaſs; for then the water or wine poured into the glaſs continues in it while the branch is filling, and till it comes to the bend or the uppermoſt part of the crane, which withal ſhould be a little lower than the upper edge of the glaſs: but after that if you continue to pour more in, it will riſe higher in the concavity of the glaſs, and not finding place for a farther aſcent into the crane by reaſon of its bending downwards, it will change its aſcent into a deſcent through the branch, and continue to deſcend and run out by the end, as long

as you continue to pour in; nay, when you have done pouring, you will ſee that all that was in the glaſs before is gone.

You may make the water run out at the lower end, though the glaſs is not filled up to the top of the crane, namely, by ſucking at the lower aperture the air contained in the crane, for then the water will neceſſarily ſucceed in the room of the air, and continue to deſcend through the branch, till the glaſs is empty, eſpecially if the orifice touches the bottom of the glaſs.

Or elſe; run the ſmall pipe perpendicular down through the glaſs; let the pipe be open at both ends, the uppermoſt of which ought to be a little lower than the brim of the glaſs, and the other end a little lower than the bottom of the glaſs. Put this ſmall pipe in another larger pipe ſtopped at the upper end, which muſt be a little higher than the end of the firſt and ſmaller pipe, and open at the lower end, which muſt touch the bottom of the glaſs if you would have all the water run out, which it will do when it riſes, for then paſſing through the orifice of the pipe, it will enter the pipe by one end and run out at the other.

SPORT CXV.

To make a coach that a man may travel in without horſes.

THE two four-wheels muſt be little, and moveable round their common Axeltree, as in the ordinary coaches and the hinder wheels muſt be large, and firmly fixed to their common axel-tree, inſomuch that the axeltree cannot move, without the wheels move along wito it.

Round the middle of the Axel-tree put a trundle-head, with ſtrong and cloſe ſpindles, and near to that fix upon the beam a notch'd wheel, the

 notches

notches of which may catch the ſpindles of the trundle-head, and ſo in turning with the handle, that wheel round its axeltree, which ought to be perpendicular to the horizon, it will turn the trundle, and with that the axeltree and the wheels, which will thereupon ſet forward the coach, without horſes or any other animal. I need not tell you that the axeltree muſt enter into the beam, in order to turn within it.

There was invented at Paris, ſome years ago, a coach or chaiſe, which a footman behind it made to go with his two feet alternately, by virtue of two little wheels hid in the box between the two hind-wheels, and made faſt to the axeltree of the coach.

In ſhort, the contrivance of the machine is this. In a roller, the two ends of which are made faſt to the box behind the chaiſe, is a pully upon which runs the rope that faſtens the end of the planks, upon which the footman puts his feet, where is a piece of wood that keeps faſt the two planks at the other end, allowing them to move up and down by the two ropes, tied to their two ends, where there are two little plates of iron which ſerve to turn the wheels that are fixed to their axeltree, which is likewiſe fixed to the two great wheels.

Thus, you will readily apprehend that the footman putting his feet alternately upon the end of each plank, one of the plates will turn out of the notch'd wheels; for example, if he leans with his foot upon one plank it deſcends and raiſes the other, which can't riſe but at the ſame time the plate of iron that enters the notches of the wheel, muſt needs make it turn with its axeltree, and conſequently the two great wheels. Then the footman, leaning upon one plank, the weight of his body will make it deſcend and raiſe the other plank, which

which turns the wheel again; and ſo the motion will be continued.

'Tis eaſy to imagine that while the two hind-wheels advance, the two fore-wheels muſt likewiſe advance; and that theſe will always advance ſtraight if the perſon that ſits in the chaiſe manages them with reins made faſt to the forebeam.

SPORT CXVI.

To make two Images, one ſhall light a candle and the other blow it out.

UPON the ſide of a wall make the figure of two images, in the mouth of each put a pipe or quill, ſo artificially that it be not perceived, in one of which place ſalt-petre very fine and dry and pulverized, and at the end ſet a little match of paper: in the other quill ſulphur beaten ſmall. Then, holding a lighted candle in your hand, ſay to one of thoſe images, by way of command, *Blow out the candle*, then lighting the paper with the candle, the ſalt-petre will blow out the candle immediately, and going to the other image (before the ſnuff of the candle be out) touch the ſulphur with it, and ſay, *Light the candle*, and it will immediately be lighted.

SPORT CXVII.

To ſeem to turn water into wine.

TAKE four beer-bowl-glaſſes, rub one in the inſide with a piece of allum; let the ſecond have a drop of vinegar in it, the third empty, and the fourth as much clean water in it as your mouth will contain: have ready in your mouth a clean rag with ground braſil tied cloſe in it, that the bulk may be no bigger than a ſmall nut, which muſt lie betwixt your hind teeth and your cheek; then take of the water out of the glaſs into your

mouth, and return it into the glaſs that hath the drop of vinegar in it, which will cauſe it to have the perfect colour of ſack; then turn it in your mouth again, and chew your rag of braſil betwixt your teeth, and ſquirt the liquor into the empty glaſs, and it will have the perfect colour and ſmell of claret, returning the braſil into its former place, take the liquor into your mouth again, and preſently ſquirt it into the glaſs you rubbed with allum, and it will have the perfect colour of mulberry wine; and ſo in many other ways, which for brevity I omit.

SPORT CXVIII.

To deceive one with ſeeming pieces of tobacco-pipe.

ROUL up a piece of white paper as hard as your lottery tickets, till it is as thick as a tobacco pipe, then faſten the outward edge with a little ſtarch or paſte, having ſo done, cut the ends even; have this in your hand, break two pieces in the ſight of the company, ſhake the three together in your hat, then caſt them upon the table, ſaying, how many pieces of pipe is there under the hat? Every one will be apt to ſay, three; lift up the hat the better to urge them, clapping it down preſently, ſaying, now I'll hold you a wager there are but two pieces of pipe under the hat; which when laid, take up the hat, and their folly will ſoon be diſcerned, by your cutting the paper with your knife.

SPORT CXIX.

To write letters ſecretly, that cannot be diſcovered.

TAKE a ſheet of white paper, and double it in the middle, then cut holes through both the half ſheets, let the holes be cut like the panes of glaſs windows, or other forms what you beſt fancy,

and

and then with a pin prick two little holes at each end and cut your paper in two halves, give one half to your friend (to whom you intend to write) the other half keep to yourſelf: now when you do write, lay your cut paper on a half ſheet of writing paper, and ſtick two pins through the two holes that it ſtir not; then through thoſe holes that you did cut, write your mind to your friend; when you have done, take off your paper with the holes again, and then write ſome other idle words both before and after your lines; but if they were written to make ſome little ſenſe, it would carry the leſs ſuſpicion; then ſeal it up and ſend it.

When your friend hath received it, he muſt lay his paper on the ſame, putting pins into the pin-holes, and then he can read nothing but your mind which you writ, for all the reſt of the lines are covered.

SPORT CXX. *Secret Writing.*

WRITE a letter (what you pleaſe) on one ſide of paper with common ink, then turn your paper, and write on the other ſide with milk, (that which you would have ſecret) and let it dry; but this muſt be written with a clean pen: Now when you would read it, hold that ſide which is written with ink to the fire, and the milky letters will then ſhew blewiſh on the other ſide, which may be perfectly diſcerned.

SPORT CXXI. *Secret Writing.*

PUT the powder of roach allum into a litttle water, and with that write upon the paper: when the letters are dry they will diſappear; but clap the paper in fair water, and the letters will look white and ſhining, the paper being a little blacked with the allum.

SPORT

SPORT CXXII. *Secret Writing:*

WRITE with the water in which ſal armoniac well pulverized is diſſolved: when the letters thus written are dry, they will diſappear, but hold them near the fire, and then they become viſible again. The ſame may be performed by writing with the juice of a lemon, or an onion.

SPORT CXXIII.

To make Glaſs-Drops.

GLaſs-Drops are thick little pieces of glaſs, made almoſt like a drop, which have a long ſlender end, which being broken at its exttemity, the drop breaks preſently with a crack, and flies into white powder and little fragments to two or three foot round.

Theſe drops, which have excited the curioſity, and perplexed the reaſon of moſt philoſophers, are made by letting a little of the melted matter of which the ordinary glaſſes are made, fall into a veſſel full of cold water; for then this melted matter which is very glutinous while 'tis red, makes a long ſtring, by which they hold the drop in the middle of the water, where it cools and hardens in a little time; after which they ſeparate the ſtring which is out of the water, ſo that the remaining part in the water does not break, commonly called a glaſs drop, to this drop there ſticks a ſmall end, part of which may be ſeparated, by making it red at the flame of a candle, without breaking the drop; nor will this drop break if you lay it upon wood, and with a hammer ſtrike upon its thickeſt part, for its external parts are very hard, and ſupport one another like a vault. And they only break upon bending the ſlender end till it breaks, by virtue of the ſpring raiſed by that effort in all its parts, which

ſhake

ſhake and tremble like an extended ſtring, put into motion by forcing it to bend; whence it comes that theſe parts do in a little time return with very great veloſity to their firſt diſpoſition; and that the parts which are leſs united, and only contiguous, as it were, diſunite and ſeparate, and that occaſions the diſunion and ſeparation of all the reſt, and their flying all about with a noiſe. See upon this head Mr. *Mariotte's* diſcourſe of the nature of the air publiſhed in 1679, in which he has, in my opinion, wrote more pertinently of this ſubject, than any one beſides.

SPORT CXXIV.

To repreſent the four elements in a vial.

THE four elements of which the author of nature has compoſed the elementary world, are the *earth*, *water*, *air*, and *Fire*; of which, the earth being the heavieſt is ſaid to have the lowermoſt ſtation in the center of the world: water being lighter covers the earth; air being lighter than water covers it; and at laſt fire the lighteſt of all ſurrounds the air. So that in this ſenſe theſe four make four concentrical orbs, the common center of which is the centre of the world.

We may repreſent the four elements in this order, in a long vial of glaſs or cryſtal, by the help of four heterogeneous liquors, that is liquors of a different ſpecific gravity, which are of ſuch qualities, that, though ſhaked together by a violent agitation, they ſoon after return to their natural ſtations, and all the particles of one and the ſame liquor unite in a ſeparate body from the reſt, the lighter giving way to the heavier.

To repreſent the earth, make uſe of crude antimony, or blue ſmalt well refined, or black ſmalt

coarſly

coarſly pounded, which by its weight will ſink to the bottom of the vial.

To repreſent water, pour upon the laſt the terreſtrious ſubſtance of the ſpirit of tartar, or calcin'd tartar, or the clean ſolution of pot-aſhes with a little roch-azur, which will give a ſea colour.

To repreſent the air, pour upon this compoſition ſpirit of wine rectified three times, till it has a colour of air, or elſe the moſt ſpirituous brandy with a little turnſol, which will give it a celeſtial blew or air colour.

To repreſent fire, pour upon all three the oil of Behn, which by its colour, likeneſs and ſubtilty, will make a pretty near reſemblance.

SPORT CXXV. *A Trick.*

TAKE a ball (or any thing elſe) in each hand, and ſtretching them as far as you can aſunder, lay a wager with any perſon, that without bringing your hands nearer together, or throwing either ball from one hand to the other, you will have them both in which hand the perſon pleaſes who lays the wager with you: which is no more than to lay one ball down upon the table, and turning yourſelf round take it up with the other hand, without bringing your hands any cloſer.

THE END.

Explanation of the technical and abſtruſe terms, uſed in the foregoing Recreations.

A

ANALOGY, the relation which one thing bears to another.

ALIQUOT, a part contained in the number preciſely ſo many times.

ALTITUDE, height. Altitude of a figure is the neareſt diſtance between the top of its baſe.

ALEMBIC, a veſſel for diſtilling.

ALUMINOUS, belonging to allum.

B

BASIS, foundation, or bottom.

BIQUADRATE, the fourth power ariſing from the multiplication of a ſquare number or quantity by itſelf.

C

CONCAVE, hollowneſs of a roundiſh body.

CONVEX, the external round part of any body oppoſite to the hollow.

COAGULATION, a curdling or thick ſubſtance.

CATAPLASM, a poultice of herbs or roots.

CONCENTRICAL, that hath one of the ſame center.

CYLINDER, a roller, or rolling ſtone.

CURVE, a crooked line.

CIRCUMVOLUTION, a rolling or turning about.

CONDENSATE, to thicken or grow thick.

CRYSTALLIZE, to ſhoot into cryſtals by being diſſolved in liquor.

CORRODE, to gnaw or fret.

CALCINE, to burn to a cinder.

CONSTRICTION, crouding the parts cloſe together.

CENTER,

CENTER, the heart, or middle point of a thing.
CEREBELLUM, the hinder part of the brain.
CONSECUTIVE, ſucceeding one another.
CUBE, a ſol d body in ſix equal ſquares.
CUBE NUMBER, is that which ariſes from the multiplication of any number by itſelf, and then by the product.
CUBE-ROOT, is the ſide of a cube-number; ſo 3 is the ſide or root of 27.

D

DEFERENS, that hath not the ſame center.
DIURNAL, daily, or belonging to the day.
DULCIFIE, to make ſweet, or ſweeten.
DESICCATION, a drying up.
DIAGONAL, a line drawn from angle to angle.
DECUPLE, ten fold.
DENOMINATOR, of a fraction, is that part of the fraction that ſtands below the line of ſeparation.
DIVIDEND, a number to be divided.
DIVISOR, the number by which the dividend is to be divided.

E

ECCENTRIC, that hath not the ſame centre.
ECLIPTIC, is a great circle of the Heavens, in which the ſun moves in its annual motion.
EFFLUX, a flowing out.
EFFLUVIUMS, ſmall particles flowing out of mixt bodies.
EBULLITION, boiling or bubling.
EFFERVENCENCY, a boiling over, or being very hot.
EXHALATIONS, fumes or vapours.
EXPONENT, a quotient ariſing when the antecedent is divided by the conſequent; and being placed over any power ſhews how many multiplications are neceſſary to produce that power.
EQUILATERAL, whoſe ſides are all equal.

F

F

FOCUS, a point where the rays meet and crofs the axis after their refraction by the glafs.

FILTRATE, to ftrain through cloth or paper.

FERMENT, to puff up or work, as ale or beer.

FLUX, floowing or loofening.

FIBRES, threads of hair like ftrings of mufcles, veins, plants, roots, &c.

G

GNOMON, the cock of a dial.

GENERATE, to engender or beget.

H

HOMOGENEOUS, of the fame kind, nature, and pvoperties.

HORIZON. the circle which bounds the fight of any perfon, who in a large plain, or in the midft of the fea, looks round about.

HUMID, damp or moift..

HYPOTHENUSE, in a right-angled triangle, is that fide which fubtends the right angle.

HARMONIC, the divifion of a line.

I

IMPREGNATE, to foak or drink in.

L

LENS, a concave or convex glafs, that is made to throw the rays of vifion into a point.

LENTICULAR, belonging to the humour of the eye.

LUTING, covering or ftopping up with loom or clay.

LONGITUDINAL, at length, or lengthways as oppofed to tranfverfe.

M

MAGNETIC, endued with the property of attracting iron to itfelf.

MEDIUM, the middle ftate.

MERIDIAN, is a great circle paffing through the poles of the world, when the fun comes to this circle,

circle, it is then mid-day or noon.

MORBIFIC, cauſing diſeaſes.

N

NUMERATOR, of a fraction, is the number fixed above the ſeparating line.

O

OCTODECUPLE, eighteen-fold.

P

PERFORATION, a boring through.

PERABOLIC, a ſolid figure, ſo called from its formation.

PRISM, a ſolid glaſs, through which the ſun's rays being tranſmitted, are refracted into the vivid colours of the rainbow.

POLYGON, having many corners.

PYRAMIDAL, in the form of a pyramid.

PENTAGON, a figure having five ſides and five angles.

PERCUSSION, a ſtriking or knocking.

PRODUCT, is the number ſought or ariſing from the multiplication of ſeveral numbers.

Q

QUOTIENT, the number that ſhews how many times the diviſor is contained in the dividend.

R

REFRACT, to break again, or reſiſt.

REFRACTION, a ſudden change of determination in a body moved.

RAREFY, to make thin.

REFRIGERATE, to refreſh or cool.

REVERBERATE, to ſtrike or beat back.

RATIO, the rate or proportion which ſeveral quantities or numbers have one to another.

RECTANGULAR, when one or more of the angles are equal.

RETROGRADE, going backwards.

S

S

STYLUS, a line whofe fhadow on the plane of the dial fhews the true hour-line, and is the upper edge of the gnomon.

SPECIFIC, a particular that diftinguifhes a thing from another of a different fpecies.

SEXTUPLE, fix-fold.

SPATULA, an inftrument for fpreading falves, &c.

SYMPATHY, a conformity in nature, a fellow-feeling.

SURSOLID, the fifth power from any given root in fpecies or numbers.

SPHERICAL, belonging to, or round like a fphere.

SEARCED, dry, or confumed.

SPONTANEOUS, acting of its own accord.

SQUARE-NUMBER, any number which is fquared or multiplied by itfelf, as 2 by 2, which is 4.

SQUARE-ROOT, the fide of a fquare-number, fo 4 is the fide of 16.

T

TRANSPIRATION, breathing of vapours through the pores.

TRANSPLANTATION, the removing things from one place to another.

TRANSVERSE, that runs acrofs.

TARIFF, a book of rates.

V

VERTICAL, a line perpendicular to the horizon.

VULNERARY, good to cure wounds.

VEGITATE, to grow, or make lively.

Thomin
communément

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